Online Appendix for “When Candidates Value Good Character: A Spatial Model with Applications to Congressional Elections”

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This appendix reports several types of supplementary analyses that are described in the paper “When Candidates Value Good Character: A Spatial Model with Applications to Congressional Elections.” We present five types of analyses in this memo. First, we present the proof of Theorem 1 in the paper (the Good Government Result). Second, we prove a theorem that extends the Good Government Result to a more general model of candidate and voter motivations, in which candidates may derive private benefits from holding office and where voters may attach more weight to one of the candidates’ character-based valence than to the other’s character-based valence (an extension that may capture scenarios where voters can more easily observe the character qualities of the incumbent). We label this more general theorem the Generalized Good Government Result. Third, we extend the Generalized Good Government Result to scenarios where the candidates have biased perceptions of their character-based valence (relative to that of their opponent), an extension that addresses the possibility that candidates systematically overestimate their own levels of competence and honesty. Fourth, we report computations on Nash equilibrium configurations of candidate strategies, for situations where both the challenger and the incumbent can shift their policy positions. These computations suggest that the Good Government dynamic typically extends to these more general scenarios. Fifth, we report additional analyses—based both on computer simulations and on empirical data for congressional elections—in which we disentangle the relative effects on challenger strategies of challengers’ versus incumbents’ character-based valence. These analyses are motivated by the intuition that voters can more easily observe incumbents’ character-based valences and so may weight incumbent character more heavily than challenger character in their voting decisions.

Proof of Theorem 1

Let voter i’s utility $U_i(j)$ for candidate j be:

$$U_i(j) = g(s_j, v_i) + V_{Si} + \alpha V_{Cj},$$

(A1)

where $g(s_j, v_i)$ represents voter i’s utility for j’s position $s_j$, $g(s_j, v_i)$ is strictly concave and peaks at voter i’s position $v_i$ and has continuous partial derivatives of order two,¹ where $V_{Si}$ and $V_{Cj}$ represent j’s strategic and character-based valence, respectively, and where $\alpha$ is a weighting parameter that denotes how much the candidates’ character-based valences influence voters’ evaluations. Similarly, let candidate R’s utility for the winning candidate w be:

$$U_R(w) = h(s_w, p_R) + V_{Cw},$$

(A2)

where $h(s_w, p_R)$, which represents R’s utility for the winning candidate’s position $s_w$, is strictly concave and peaks at candidate R’s preferred position $p_R$ and has continuous partial derivatives of

¹ Recall that this implies that there is a strictly concave function, say $\bar{g}$, of a single variable such that $\bar{g}(s_j - v_i) = g(s_j, v_i)$ for all $s_j$ and $v_i$, and $\bar{g}$ has a continuous derivative.
order two, and \( V_{Cw} \) represents the winning candidate’s character-based valence. Let the location of the median voter position be \( M = m + \varepsilon_M \), where \( m \) is the measured component of the median voter position and \( \varepsilon_M \) is the random component that follows a continuous distribution that is centered on zero with cumulative distribution function \( F \) and probability density function \( f \).

**Theorem 1 (The Good Government Result).** Assume that the incumbent \( D \)’s policy strategy \( s_D \) is fixed, that the challenger \( R \)’s utility function is given by equation (A2), that voters’ utility functions are given by equation A1, and that the probability density function for the median voter position has a continuous derivative. Also assume that \( R \)’s optimal position \( s_R^* \) lies strictly between \( s_D \) and \( P_R \), where \( P_R \) is \( R \)’s sincere Left-Right preference. Then there exists \( \beta > 0 \) such that if \( 0 < \alpha < \beta \), then as \( R \)’s character-based valence improves (declines) relative to that of \( D \), \( R \) is motivated to shift towards (away from) \( s_D \).

**Proof.** Based on equation (A2), candidate \( R \)’s expected utility \( EU(R) \) is

\[
EU(R) = P(R)U_R(R) + [1 - P(R)]U_R(D) = P(R)[h(s_R, p_R) + V_{CR}] + [1 - P(R)][h(s_D, p_R) + V_{CD}]
\]

where \( P(R) \) is \( R \)’s probability of being elected and we write \( V = (V_{CR} - V_{CD}) \) for simplicity. It follows that

\[
\frac{\partial EU(R)}{\partial s_R} = \frac{\partial P(R)}{\partial s_R} [h(s_R, p_R) - h(s_D, p_R) + V] + P(R) \frac{\partial h(s_R, p_R)}{\partial s_R} . \quad (A3)
\]

When \( R \) locates at his optimal position \( s_R^* \) then \( \frac{\partial EU(R)}{\partial s_R} = 0 \). The strategy of the proof of Theorem 1 is to show that as \( R \)’s character-based valence improves relative to that of \( D \), i.e. as \( V \) increases, then, for \( s_D < s_R < P_R \), \( \frac{\partial EU(R)}{\partial s_R} \) decreases.\(^2\) This will demonstrate that when \( R \) locates at \( s_R^* \) for a fixed value of \( V \), then an increase in \( V \) implies that \( \frac{\partial EU(R)}{\partial s_R} < 0 \) when evaluated at \( s_R^* \), which implies in turn that \( R \) can increase her expected utility by shifting unilaterally to the left, in the direction of \( s_D \). Thus,

\[
\frac{\partial}{\partial V} \left( \frac{\partial EU(R)}{\partial s_R} \right) = \frac{\partial}{\partial V} \left( \frac{\partial P(R)}{\partial s_R} \right) [h(s_R, p_R) - h(s_D, p_R) + V] + \left( \frac{\partial P(R)}{\partial V} \right) \frac{\partial h(s_R, p_R)}{\partial s_R} . \quad (A4)
\]

Next we show that as \( \alpha \), the weighting parameter in equation A1 that denotes how much the candidates’ character-based valence \( V \) influences voters’ evaluations, approaches zero, the terms \( \frac{\partial}{\partial V} \left( \frac{\partial P(R)}{\partial s_R} \right) \) and \( \frac{\partial P(R)}{\partial V} \) in equation (A4) above approach zero. From this it will follow that:

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\(^2\) The logic of the proof is identical if we analyze the scenario where \( P_R < s_R < s_D \).
Let $c$ denote the cutpoint where voters are indifferent between $D$ and $R$, i.e., for which
\[ g(s_D, c) - g(s_R, c) = (V_{sR} - V_{sD}) + \alpha V. \] (A6)

To show that the terms \( \frac{\partial P(R)}{\partial V} \) and \( \frac{\partial (P(R))}{\partial \alpha} \) approach zero as $\alpha \to 0$, note that for $s_D < s_R$,
\[ \frac{\partial P(R)}{\partial V} = \frac{\partial [1 - F(c)]}{\partial V} = -f(c)\frac{\partial c / \partial V}{\partial V}, \] (A7)
and
\[ \frac{\partial (P(R))}{\partial s_R} = \frac{\partial (P(R))}{\partial s_R} = \frac{\partial (P(R))}{\partial s_R} = -f(c)\frac{\partial c / \partial V}{\partial V} - f'(c)\frac{\partial c / \partial s_R}{\partial V}. \] (A8)

To see that \( \frac{\partial P(R)}{\partial V} \) approaches zero as $\alpha \to 0$, we first note that by implicit differentiation of equation A6, we obtain $\frac{\partial c/\partial V}{\partial V}$
\[ \frac{\partial c}{\partial V} = \frac{\alpha}{\partial g(s_D, c)/\partial c - \partial g(s_R, c)/\partial c}. \] 4
Defining $k(x) = [\partial g(s_D, x)/\partial x - \partial g(s_R, x)/\partial x]$, we have by strict concavity of $g$ that $k(x) > 0$ on the interval $[s_D, p_R]$, so that by continuity of $k$, $k(x)$ is bounded away from zero on this closed interval, i.e., there exists $\varepsilon > 0$ such that $k(x) \geq \varepsilon$ for all $x$ in the interval. It follows that $\partial c / \partial V$ equals $\alpha$ times a bounded function, so that there exists $M > 0$ such that $\partial c / \partial V \leq \alpha M$ throughout the interval. In turn, since $f$ is continuous and hence bounded, then by equation A7, \( \frac{\partial P(R)}{\partial V} \) equals $\alpha$ times a bounded function on the interval $[s_D, p_R]$ and hence approaches zero as $\alpha \to 0$.

Next, to show that \( \frac{\partial (P(R))}{\partial s_R} \) approaches zero as $\alpha \to 0$, we first show that $\partial c / \partial s_R$ is bounded.

Recalling that $\overline{g}(s_j - v_j) = g(s_j, v_j)$, we have
\[ \overline{g}(s_D - c) - \overline{g}(s_R - c) = V_{sR} - V_{sD} + \alpha V \]
at equilibrium. By implicit differentiation, we obtain

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3 The conditions that we assume here, i.e., that voters’ policy loss functions are strictly concave and voter utility is an additively separable function of the candidates’ valence characteristics and their policy positions, is sufficient to guarantee that a cut-point exists whenever the candidates announce divergent policy positions (see Groseclose 2001, 883).

4 To derive this, consider $c$ as an (implicit) function of $V$ and differentiate equation A6 with respect to $V$ using the chain rule, obtaining \[ \frac{\partial g(s_D, c)/\partial c}{\partial V} \frac{\partial c}{\partial V} - \frac{\partial g(s_R, c)/\partial c}{\partial V} \frac{\partial c}{\partial V} = \alpha. \] Then solve for $\frac{\partial c/\partial V}$. 

3
\[
\frac{\partial c}{\partial s_R} = \frac{\tilde{g}'(s_R - c)}{\tilde{g}'(s_R - c) - \tilde{g}'(s_D - c)}.
\]

By the mean value theorem, \( \tilde{g}'(s_R - c) - \tilde{g}'(s_D - c) = \tilde{g}''(x)(s_R - s_D) \) for some \( x \) between \( s_D - c \) and \( s_R - c \). As long as \( s_R \) remains in a neighborhood of \( s_R^* \) (away from \( s_D \)), the denominator of \( \frac{\partial c}{\partial s_R} \) is bounded away from zero because \( \tilde{g}'' \) is bounded away from zero by assumption and continuity. Since, also, \( \tilde{g}' \) is bounded (by continuity of \( \tilde{g}' \)), we conclude that \( \frac{\partial c}{\partial s_R} \) is a bounded function. Finally, we show that \( \frac{\partial}{\partial s_R} \left[ \frac{\partial c}{\partial V} \right] \) is less than \( \alpha \) times a bounded function.

Differentiating equation A6 implicitly with respect to \( s_R \) yields

\[
\frac{\partial}{\partial s_R} \left[ \frac{\partial c}{\partial V} \right] = \frac{\frac{\partial}{\partial s_R} \left( \frac{\partial g(s_D, c)}{\partial c} \right) + \frac{\partial}{\partial s_R} \left( \frac{\partial g(s_R, c)}{\partial c} \right)}{\frac{\partial g(s_D, c)}{\partial c} - \frac{\partial g(s_R, c)}{\partial c}}.
\]

As we have seen, the denominator of this expression is bounded from zero; the term \( \frac{\partial}{\partial s_R} \left[ \frac{\partial c}{\partial V} \right] \) is equal to \( \alpha \) times a bounded function, while the second factor in the numerator is bounded because of the continuity of the partial derivatives. Furthermore, both \( f \) and \( f'' \) are bounded because they are continuous on a closed interval. Thus each term on the right-hand side of equation (A8) is the product of bounded factors one of which is equal to \( \alpha \) times a bounded function. This completes the argument that \( \frac{\partial P(R)}{\partial s_R} \) and \( \frac{\partial P(R)}{\partial V} \) both converge to zero when \( \alpha \) approaches zero and thus that statement A5 holds.

To show that for \( \alpha \to 0 \), \( \frac{\partial P(R)}{\partial s_R} < 0 \) when evaluated at \( s_R^* \), we employ the following argument. Recalling that \( c \) is the cutpoint where voters are indifferent between \( D \) and \( R \), then, for \( s_D < s_R^* \), \( P(R) = [1 - F(c)] \) where \( F(c) \) represents the cumulative distribution of the probability function on the median voter position evaluated at \( c \), so that

\[
\left( \frac{\partial P(R)}{\partial s_R} \right) = - \frac{\partial c}{\partial s_R} f(c) \quad , \tag{A9}
\]

where \( f(c) \) is the probability density function on the median voter position evaluated at \( c \).

Equations (A5) and (A9) imply that when \( \alpha \to 0 \) then \( \frac{\partial}{\partial s_R} \left[ \frac{\partial EU(R)}{\partial V} \right] \to \left( - \frac{\partial c}{\partial s_R} f(c) \right) \), which

implies that for sufficiently small \( \alpha \), \( - \frac{\partial}{\partial s_R} \left[ \frac{\partial EU(R)}{\partial V} \right] \) will have the same sign as \( - \frac{\partial c}{\partial s_R} f(c) \). Now, Theorem 1 applies to the situation where \( R \)’s optimal position \( s_R^* \) lies strictly between \( s_D \) and \( p_R \).
But, note that \( s_D < s_R^* < p_R \) implies that the cutpoint \( c \) must be located strictly to the left of \( s_R^* \), i.e., \( s_D < s_R^* < p_R \) implies that \( c < s_R^* \). To see this, note that for a fixed value of \( V \), \( R \)’s optimal position \( s_R^* \) must be such that\[\frac{\partial EU(R)}{\partial s_R} = 0 \] when evaluated at \( s_R^* \), and by equation A3 this implies that\[\frac{\partial P(R)}{\partial s_R} \left[ h(s_R, p_R) - h(s_D, p_R) + (V_{CR} - V_{CD}) \right] = -P(R) \frac{\partial h(s_R, p_R)}{\partial s_R}. \] (A10)

Equation (A10) implies that \( \frac{\partial P(R)}{\partial s_R} < 0 \) when evaluated at \( s_R^* \), since \(-P(R) \frac{\partial h(s_R, p_R)}{\partial s_R} \) is strictly negative\(^5\) when evaluated at \( s_D < s_R^* < p_R \), while \[ h(s_R, p_R) - h(s_D, p_R) + (V_{CR} - V_{CD}) \] is strictly positive when evaluated at \( s_R^* \) (else \( R \) would not even contest the election). But, \( \frac{\partial P(R)}{\partial s_R} < 0 \) implies that the cutpoint \( c \) must be located strictly to the left of \( s_R^* \), i.e., \( \frac{\partial P(R)}{\partial s_R} < 0 \) implies \( c < s_R^* \).

Now, when \( c < s_R^* \) then \( \frac{\partial c}{\partial s_R} > 0 \), i.e., when the cutpoint is located to the left of \( R \)’s position, then as \( R \) shifts unilaterally to the right the cutpoint \( c \) shifts to the right as well. But from this it follows that\[\alpha \to 0 \Rightarrow \frac{\partial}{\partial V} \left( \frac{\partial EU(R)}{\partial s_R} \right) \Rightarrow \frac{\partial}{\partial V} \left( \frac{\partial P(R)}{\partial s_R} \right) \Rightarrow \frac{\partial}{\partial V} \left( \frac{\partial EU(R)}{\partial s_R} \right) \Rightarrow \frac{\partial}{\partial V} \left( \frac{\partial c}{\partial s_R} f(c) \right) < 0 \], when evaluated at \( s_R^* \).

We have succeeded in proving what we set out to prove, namely that as \( R \)’s character-based valence improves relative to \( D \), i.e., as \( V = (V_{CR} - V_{CD}) \) increases, then when \( R \) is located at his equilibrium position \( s_R^* \), then \( \frac{\partial}{\partial V} \left( \frac{\partial EU(R)}{\partial s_R} \right) \Rightarrow \frac{\partial}{\partial s_R} f(c) < 0 \) for \( \alpha \to 0 \). This result, combined with the fact that for a fixed value of \( V \), \( \frac{\partial EU(R)}{\partial s_R} = 0 \) when evaluated at \( s_R^* \), implies in turn that as \( V = (V_{CR} - V_{CD}) \) increases, then \( \frac{\partial EU(R)}{\partial s_R} < 0 \) when evaluated at \( s_R^* \). But this implies that as \( R \)’s character-based valence improves, \( R \) can improve his expected utility by shifting unilaterally to the left, towards \( D \)’s position \( s_D \).

Generalizing the Good Government Result: Extensions to More General Models of Candidate and Voter Motivations

\(^5\) This follows because \( h \) is strictly concave and hence strictly increasing to the left of \( p_R \).
The Good Government Result (Theorem 1 in the paper) applies to a specification of candidate motivations where the candidates are entirely concerned with policy outputs along with the winning candidate’s character-based valence (see equation (2) in the paper). However, candidates may also value holding office independently of policy- or character-seeking motivations, i.e., they may also be office-seeking as in the original Downsian model. With respect to the electorate, the specification of voter motivations that we present in the paper (see equation (1) in the paper) assumes that voters attach equal weights to the character traits of the incumbent and the challenger. Intuitively, however, it seems plausible that voters can more easily obtain information about the incumbent, so that the incumbent’s character traits may disproportionately influence voters’ decisions and hence the election outcome. To capture these considerations we modify candidate R’s utility for the winning candidate, \( U_R(w) \), to include office-seeking motivations by including a term \( O(R) \), that equals \( \varepsilon \) if \( R \) wins the election and zero otherwise:

\[
U_R(w) = h(s_w, p_R) + V_{CW} + O(R),
\]

(S1)

where \( h(s_w, p_R) \) denotes \( R \)’s utility for the winning candidate’s policy strategy \( s_w \) and \( V_{CW} \) denotes the winning candidate’s character-based valence. We modify the voter’s utility calculus to include candidate-specific character weights \( \alpha_D \) and \( \alpha_R \), so that voter \( i \)’s utilities for the candidates are

\[
U_i(D) = g(s_D, v_i) + V_{SD} + \alpha_D V_{CD}, \quad \text{and}
\]

\[
U_i(R) = g(s_R, v_i) + V_{SR} + \alpha_R V_{CR},
\]

(S2)

where \( \alpha_D \) and \( \alpha_R \) denote the non-negative weights that voters attach to the character-based valences of candidates \( D \) and \( R \), respectively, while \( V_{SD} \) and \( V_{SR} \) represent the candidates’ strategic valences and \( g(s_D, v_i) \) and \( g(s_R, v_i) \) denote the voter \( i \)’s utilities for the candidates’ policy strategies \( s_D \) and \( s_R \). To the extent that voters possess more information about the incumbent \( D \)’s character traits we might expect \( \alpha_D > \alpha_R \), i.e., that voters will weight the incumbent’s character-based valence more heavily than the challenger’s character-based valence in their voting decisions.

Our Generalized Good Government Result extends Theorem 1 in the paper to the more general model outlined above, in which the candidates may have office-seeking motivations and voters may attach different weights to the character traits of incumbents and challengers:

**Theorem S1 (The Generalized Good Government Result).** Assume that the incumbent \( D \)’s policy strategy \( s_D \) is fixed, that the challenger \( R \)’s utility function is given by equation S1, i.e. \( R \) is motivated by a combination of policy-, character-, and office-seeking objectives, and that voters’ utility functions are given by equation S2, i.e. voters may attach different weights to the character-based valences of the candidates \( D \) and \( R \). Then, for the conditions outlined in Theorem 1 in the paper:

1) (The challenger character dynamic): There exists \( \chi \) such that if \( 0 < \alpha_R < \chi \), then as \( R \’s \) character-based valence \( V_{CR} \) improves (declines), \( R \) is motivated to shift towards (away from) \( m \), and

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6 The private benefits that office-holders receive include such factors as salary, fame, prestige, travel, and (in some cases) improved prospects for high-paying jobs in private industry upon leaving office.
2) (The incumbent character dynamic): There exists \( \delta \) such that if \( 0 < \alpha_D < \delta \), then as \( D \)'s character-based valence \( V_{CD} \) improves (declines), \( R \) is motivated to shift away from (towards) \( m \),

where \( m \) represents the median point of the probability distribution on the median voter position.

In words, Part 1 the Generalized Good Government Result states that, for the more general model of candidate and voter motivations given by equations (S1-S2), then, provided that the challenger \( R \)'s character-based valence does not weigh too heavily in voters' decisions, the better \( R \)'s character-based valence the more moderate \( R \)'s optimal position (the challenger character dynamic). And, when the incumbent \( D \)'s character-based valence does not weigh too heavily in voters’ decisions, then the better \( D \)'s character-based valence the more extreme \( R \)'s optimal position (the incumbent character dynamic). The theorem is relevant to situations where voters attach different weights to the character traits of incumbents and challengers—a scenario that appears especially plausible in the scenarios we are modeling, namely those where a challenger competes against an incumbent whose character traits may be well-known to voters.

**Sketch of the Proof of Theorem S1.**

Based on equation (S1), candidate \( R \)'s expected utility \( EU(R) \) is

\[
EU(R) = P(R)U_R(R) + [1 - P(R)]U_R(D) \\
= P(R)[h(s_R, p_R) + V_{CR} + \varepsilon] + [1 - P(R)][h(s_D, p_R) + V_{CD}] \\
= P(R)[h(s_R, p_R) - h(s_D, p_R) + (V_{CR} - V_{CD}) + \varepsilon] + [h(s_D, p_R) + V_{CD}],
\]

where \( P(R) \) is \( R \)'s probability of being elected. Note that this expression for \( EU(R) \) is identical to that given in the proof of Theorem 1 in the paper except for the inclusion of the additional term \( \varepsilon \), which represents the private benefits \( R \) receives from holding office in the event she is elected.

It follows that

\[
\frac{\partial EU(R)}{\partial s_R} = \frac{\partial P(R)}{\partial s_R}[h(s_R, p_R) - h(s_D, p_R) + (V_{CR} - V_{CD}) + \varepsilon] + P(R)\frac{\partial h(s_R, p_R)}{\partial s_R}, \quad (S3)
\]

which is identical to equation (A3) in the appendix of the paper, except for the inclusion of the additional term \( \varepsilon \).

The logic of the remainder of the proof of Theorem S1 is very similar to the proof of Theorem 1 in the paper. In order to prove the first part of Theorem S1 (the challenger character dynamic), we note that when \( R \) locates at her optimal position \( s_R^* \) then \( \frac{\partial EU(R)}{\partial s_R} = 0 \), and we then show that, for a sufficiently small value of the character-weight \( \alpha_R \), then as \( R \)'s character-based valence \( V_{CR} \) increases \( \frac{\partial EU(R)}{\partial s_R} \) decreases. This will demonstrate—again for a sufficiently small value of \( \alpha_R \)—that when \( R \) locates at \( s_R^* \) for a fixed value of \( V_{CR} \), an increase in \( V_{CR} \) implies that \( \frac{\partial EU(R)}{\partial s_R} < 0 \) when evaluated at \( s_R^* \), which implies in turn that \( R \) can increase her expected utility by shifting unilaterally to the left, in the direction of \( m \).

Equation (S3) implies that
Now, note that as $\alpha_R$, the weighting parameter in equation (S1) that denotes how much the candidate $R$’s character-based valence $V_{CR}$ influences voters’ evaluations, approaches zero, the terms $\frac{\partial (\partial P(R) / \partial V_{CR})}{\partial V_{CR}}$ and $\frac{\partial (\partial P(R) / \partial V_{CR})}{\partial V_{CR}}$ in equation (S4) above approach zero. It follows that:

$$\alpha_R \rightarrow 0 \Rightarrow \frac{\partial (\partial EU(R) / \partial S_R)}{\partial V_{CR}} \rightarrow \frac{\partial (\partial P(R) / \partial S_R)}{\partial V_{CR}}.$$ (S5)

The remainder of the proof of the first part of Theorem S1 (the challenger character dynamic) employs a logic that is identical to that used in the proof of Theorem 1 in the paper. We can use an identical approach to prove the second part of Theorem S1 (the incumbent character dynamic).

Extending the Generalized Good Government Result to Situations where Challengers hold Biased Perceptions of the Candidates’ Character-Based Valences

It seems plausible that real-world candidates for office systematically overestimate their own character-based valence qualities (competence, honesty, diligence), relative to those of their opponents. To capture this possibility, denote $p(V_{CR})$ and $q(V_{CD})$ as the challenger $R$’s perceptions of her own character-based valence $V_{CR}$ and of the incumbent’s character-based valence $V_{CD}$, respectively, and assume that $\frac{\partial p(V_{CR})}{\partial V_{CR}} > 0$ and $\frac{\partial q(V_{CD})}{\partial V_{CD}} > 0$, i.e., candidates’ perceptions of their own (and their opponent’s) character increase with increases in actual character-based valence. Denote $PP(R)$ as candidate $R$’s perception of her election probability, which is a function of $p(V_{CR})$ and $q(V_{CD})$, and denote $PEU(R)$ as candidate $R$’s perception of her expected utility, which is also a function of $p(V_{CR})$ and $q(V_{CD})$. (Note that $PP(R)$ and $PEU(R)$ may differ from $R$’s actual election probability $P(R)$ and expected utility $EU(R)$, because $R$’s perceptions of the candidates’ character-based valences may diverge from voters’ perceptions of these valences.)

To see that the Generalized Good Government Result (Theorem S1) can be extended to the scenario described above, note that the derivative of candidate $R$’s perceived expected utility $PEU(R)$, with respect to $R$’s policy strategy $s_R^*$, is given by the following function:

$$\frac{\partial PEU(R)}{\partial S_R} = \frac{\partial PP(R)}{\partial S_R} [h(s_R, p_R) - h(s_D, p_R) + [p(V_{CR}) - q(V_{CD})] + \varepsilon] + PP(R) \frac{\partial h(s_R, p_R)}{\partial S_R},$$

where $h(s_R, p_R)$ represents the utility function, $s_R$ represents the policy strategy, and $p(V_{CR})$ and $q(V_{CD})$ are the candidate’s perceptions of their own and their opponent’s character-based valences.
which is identical to equation (S3) above, except that we have substituted \( PEU (R) \) for \( EU (R) \), \( PP (R) \) for \( P(R) \), and we have substituted \([ p(V_{CR}) - q(V_{CD})] \) for \((V_{CR} - V_{CD})\). The logic of the proof that Theorem S1 extends to this scenario, in which candidates’ may have biased perceptions of character-based valence, is identical to that sketched for Theorem S1. It follows that the Good Government Result that we prove in the paper, which applies to scenarios where the challenger accurately perceives her own (and her opponent’s) character-based valence, extends to a more general model where the challenger may have biased perceptions of candidate character. All that is required is that the challenger’s perception of a focal candidate’s character-based valence increases (decreases) in response to increases (decreases) in the focal candidate’s actual character-based valence, a condition that appears plausible for real-world politicians. The result applies, for instance, to scenarios where the challenger inflates her estimate of her relative character-based valence by some fixed amount, i.e., where, and \( \beta > 0 \).

Computation on Nash Equilibrium Configurations of Candidate Strategies

The theoretical and computational results we report in the paper apply to scenarios where the incumbent \( D \)’s policy strategy is fixed, for the duration of the campaign, by her previous policy behavior in office. While we believe that this is a reasonable assumption, the question arises: does a Nash equilibrium in candidate strategies\(^8\) typically exist for scenarios where both candidates have the leeway to shift their policy positions, and if so, what are the characteristics of these equilibria?

We have been unable to derive theoretical results on Nash equilibrium for candidates who value good government and policy outputs. However, to address the above questions we performed computations on scenarios where both the challenger \( R \) and the incumbent \( D \) were free to shift their policy positions, and where we systematically varied the candidates’ relative character-based valences \((V_{CR} - V_{CD})\), along with the weight \( \alpha \) that voters attached to candidate character. These computations (described below) revealed two interesting patterns:

1) For every scenario that we investigated, we located a single Nash equilibrium configuration in candidate strategies.

2) For the challenging candidate \( R \), the pattern of \( R \)’s computed equilibrium strategies \( s_R^* \) was consistent with the Good Government Result. Namely we found that as \( R \)’s character-based valence improved relative to \( D \), i.e., as \((V_{CR} - V_{CD})\) increased, then, ceterus paribus, \( R \)’s equilibrium strategy \( s_R^* \) became more moderate.

The first pattern suggests that for realistic scenarios in our model, a unique equilibrium configuration in candidates’ policy strategies is likely to exist. The second pattern suggests that the Good Government dynamic—whereby challenging candidates moderate their positions in response to improvements in their character-based valence vis-à-vis the incumbent—extends to scenarios where we relax the assumption that the incumbent candidate’s policy is fixed.

Description of the computations. We computed Nash equilibrium configurations\(^9\) \((s_D^*, s_R^*)\) for candidates \( D \) and \( R \), for scenarios where their sincerely preferred Left-Right

---

\(^8\) A Nash equilibrium is a configuration of policy strategies such that neither candidate has incentives to unilaterally shift her strategy, given the strategy of her opponent.

\(^9\) We located Nash equilibrium configurations by successively adjusting the candidates’ policy strategies, with each candidate’s optimal position selected at each iteration from the set of 1201 positions \{1.000, 1.005, 1.010, ..., 7.000\}, along the 1–7 policy scale. The candidates
positions were set to $p_D=2$, $p_R=6$; candidates and voters had quadratic policy losses; the candidates’ relative strategic valence was set to $[V_{SR} - V_{SD}] = -2$ (i.e., we assumed that the incumbent $D$ benefitted from strategic advantages); and, uncertainty about the location of the median voter position was captured by a normal probability distribution centered on 4.0 with standard deviation $= 0.5$. We performed computations where we varied the candidates’ relative character-based valence $(V_{CR} - V_{CD})$ over the interval $[-2, +2]$, and we also varied the weight $\alpha$ that voters attached to the candidates’ character-based valence over the interval $[0, 1]$. Figure S4 displays the computed equilibrium configurations. Figure S4A displays computations for scenarios where voters attached no weight to the candidates’ character-based valence ($\alpha=0$); Figure S4B displays computations when voters attached half as much weight to the candidates’ character valence as the candidates did themselves ($\alpha=0.5$); Figure S4C displays computations where voters and candidates attached equal weights to the candidates’ character traits ($\alpha=1$). For all three scenarios, we see that as $R$’s character-based valence improves relative to the incumbent $D$, i.e., as $(V_{CR} - V_{CD})$ increases, $R$’s equilibrium strategy $s_R^*$ moderates by shifting closer to 4.0 (the center of the probability distribution on the median voter position). This pattern is consistent with the Good Government dynamic we argue for in the paper, that challenging candidates moderate their policy strategies as their character-based valence improves relative to the incumbent. Indeed, these computations suggest that the Good Government dynamic is even stronger when the incumbent has the leeway to shift her policies (as in our Nash equilibrium computations) than when the incumbent’s position is fixed (as we assume in the paper). To see this, note that in the Nash equilibrium configurations illustrated in Figure S4 the Good Government dynamic applies over the full range of investigated values of $(V_{CR} - V_{CD})$, the candidates’ relative character valence, and for all values of $\alpha$, the weight that voters attach to candidate character. By contrast, Figure 2 in the paper—which reports computations for ranges of values for the parameters $p_D$, $p_R$, $(V_{SR} - V_{SD})$, $(V_{CR} - V_{CD})$, and $\alpha$ that are identical to the ranges we use for the Nash equilibrium computations reported above—shows that when the incumbent $D$’s position is fixed at $s_D = 2.5$, then the Good Government dynamic breaks down when $\alpha = 1$, i.e., when voters weight candidate character as heavily as the candidates do themselves. These computations on Nash equilibrium thereby suggest that the Good Government dynamic on challenger strategies will typically be more pronounced in situations where the incumbent can shift her policy strategy, than in situations where the incumbent’s position is fixed.

Typically converged to an equilibrium configuration in less than 10 iterations. For each scenario that we examined we initially located the candidates at the starting points $(s_D = 2, s_R = 6)$; we note that we performed a limited number of computations where we initially located both candidates at $(s_D = s_R = 4)$, and that these computations generated identical equilibrium configurations to those where the starting points were $(s_D = 2, s_R = 6)$.10 By contrast, the computations on Nash equilibrium pictured in Figure S4 suggest that, in scenarios where the incumbent has the leeway to shift her policy position, the Good Government dynamic is less pronounced with respect to the incumbent $D$’s equilibrium strategies $s_D^*$ than it is with respect to the challenger $R$’s equilibrium strategy $s_R^*$. Figure S4A, which displays computations for scenarios where voters attach no weight to the candidates’ character-based valence ($\alpha=0$), shows that the Good Government dynamic applies to the
Disentangling the Strategic Effects of Challengers’ and Incumbents’ Character-Based Valence: Simulation and Empirical Analyses

As discussed above, real-world voters in congressional elections plausibly weigh the incumbent’s character-based valence more heavily than they weigh the challenger’s character-based valence, because they can more easily observe the former than the latter. This is important because the theoretical and simulation-based results presented earlier in this memo (and in the paper) suggest that the good government effect we have identified becomes more pronounced as voters attach less weight to the candidates’ character-based valences—which implies that, to the extent that voters attach disproportionate weight to the incumbent’s character, challenging candidates should display stronger tendencies to moderate their policies as their own character-based valence increases than as the incumbent’s character-based valence declines. That is, we should expect that the challenger’s character dynamic given in Theorem S1 (the Generalized Good Government Result) should be stronger than the incumbent’s character dynamic. These considerations suggest the following hypothesis:

The Challenger’s Character Hypothesis: Challenging candidates display stronger tendencies to moderate their policy strategies as their own character-based valence increases, than as the incumbent’s character-based valence declines.

Simulation Analyses with Candidate-Specific Character Salience Parameters

To evaluate the Challenger’s Character Hypothesis we first used computer simulation to reestimate the parameters of models that were identical to the ones given in the section on simulation analysis in the paper, except that in these new models we replaced the variable \([V_{cr} - V_{cd}]\)—defined as the difference between the character-based valences of the challenger and the incumbent—with the variables \(V_{cr}\) (the challenger’s character-based valence) and \(V_{cd}\) (the incumbent’s character-based valence). For these simulations we specified voter utility functions including candidate-specific character-based valence parameters \(\alpha_D\) and \(\alpha_R\) for the candidates R and D (as in equation (S2) above). For the simulations we assumed that voters and candidates had quadratic policy losses, so that voter \(i\)’s utilities \(U_i(D)\) and \(U_i(R)\) for the candidates D and R were:

\[
U_i(D) = -(v_i - s_D)^2 + V_{SD} + \alpha_D V_{CD},
\]

\[
U_i(R) = -(v_i - s_R)^2 + V_{SR} + \alpha_R V_{CR},
\]

while candidate R’s utility for the winning candidate was given by equation (2) in our paper:

\[
U_R(w) = h(s_w, p_R) + V_{Cw}.
\]

incumbent D, in that as D’s character-based valence deteriorates relative to R, i.e., as \([V_{cr} - V_{cd}]\) increases, D’s equilibrium strategy \(s_D^*\) shifts farther away from 4.0 (the center of the probability distribution on the median voter position). However in the computations where voters weigh the candidates’ character-based valences, i.e., for \(\alpha=0.5, \alpha=1\), there is no systematic relationship between \([V_{cr} - V_{cd}]\) and D’s equilibrium position. This pattern on the incumbent’s equilibrium position thereby resembles the pattern we identified in the paper, with respect to challenging candidates who face an incumbent whose position was fixed (see Figure 2 in the paper): namely, that the Good Government dynamic applies when voters do not weigh candidate character but that this dynamic may break down when character matters to voters.
For this scenario we simulated 1,000 elections in which parameters were chosen randomly from a parameter space. Specifically, we computed the challenger R’s optimal position \( s_R^* \) for simulated elections where R’s and D’s strategic valences \( V_{sr} \) and \( V_{sd} \), and their character-based valences \( V_{cd} \) and \( V_{cr} \), were each chosen independently from uniform distributions on the interval \([-1.0, 1.0]\); the incumbent D’s fixed position \( s_D \) was chosen from a uniform distribution on the interval \([1.0, 4.0]\); and the challenger’s sincere policy preference was set to \( p_R = 6 \). With respect to character-based valence, we assumed that the weighting parameters \( \alpha_D \) and \( \alpha_R \) for the effect of the candidates’ character-based valences on voter utility were each chosen independently from a uniform distribution on the interval \([0.0, 1.0]\). Uncertainty about the median voter location was represented by a normal distribution centered on 4.0 with standard deviation=0.5, as in the simulations reported in the paper. The challenger R’s optimal strategies \( s_R^* \) were regressed on eight independent variables: R’s relative strategic valence \( [V_{sr} - V_{sd}] \); the incumbent’s position \( s_D \); the character-based salience parameter with respect to R, \( \alpha_R \); R’s character-based valence \( V_{cr} \); a variable that interacted \( \alpha_R \) with R’s character-based valence, \( [\alpha \times V_{cr}] \); and, the variables \( \alpha_D, V_{cd} \), and \( [\alpha \times V_{cd}] \), that relate to the character-based salience parameter \( \alpha_D \) and the quality of candidate D’s character-based valence \( V_{cd} \). The regression produced the following parameter estimates (see column 1 of Table S1, Part B):

\[
s_R^* = 5.030 + 0.079[V_{sr} - V_{sd}] - 0.134s_D + 0.002\alpha_R - 0.040V_{cr} + 0.033[\alpha \times V_{cr}]
\quad + 0.012\alpha_D + 0.046V_{cd} - 0.105[\alpha \times V_{cd}].
\]

These coefficients were statistically significant at the 0.05 level except for the coefficients of \( \alpha_D \) and those of the two variables involving \( \alpha_R \) (see Table S1, Part B). The residuals-by-predicted plot for this regression reveals lack of variance stability, which is ameliorated by regressing separately for fixed values of the incumbent strategy \( s_D \). For \( s_D = 2 \), for example, the value of adjusted \( R^2 \) is 0.996 and all coefficients are significant at the 0.001 level except that for \( \alpha_D \) (see column 2 of Table S1, Part B).

It follows from the parameter estimates in column 1 of Table S1, Part B (in which \( s_D \) is allowed to vary) that a one unit increase in the challenger’s character-based valence corresponds to a movement of the challenger’s strategy of \((-0.040 + 0.033\alpha_R)\), a quantity that remains negative as \( \alpha \) varies from 0 to 1, indicating that the challenger moderates as \( v_{cr} \) increases, whatever the value of \( \alpha_R \). On the other hand, when the incumbent’s character-based valence declines by one unit, the challenger’s strategy moves by \((-0.046 + 0.105\alpha_D) \) units—a quantity that is negative for \( \alpha_D < 0.44 \), but positive otherwise. Hence the overall effect on challenger strategy of increases in \( V_{cr} \) and decreases in \( V_{cd} \) tends to be moderation for smaller values of \( \alpha_R \) and \( \alpha_D \), but this effect may reverse if \( \alpha_D \) is sufficiently large. Except for very small values of \( \alpha_D \), however, the challenger’s strategy moderates more for an increase in her own character-
based valence than that of the incumbent,\textsuperscript{11} thus supporting the Challenger’s Character Hypothesis.

**Empirical Analyses with Candidate-Specific Character Salience Parameters**

To evaluate the Challenger’s Character Hypothesis empirically, we reestimated the parameters of models that were identical to the ones introduced in Table 1 in the paper, except that in these supplementary models we replaced the variable [Challenger’s relative character valence]—defined as the difference between the character-based valences of the challenger and the incumbent—with the variables [Challenger’s character-based valence] and [Incumbent’s character-based valence]. To the extent that we estimate a negative and statistically significant coefficient on the [Challenger’s character-based valence] variable—indicating that challengers tend to moderate their ideological positions as their character-based valence improves—and that the magnitude of this coefficient estimate exceeds the estimate on the [Incumbent’s character-based valence] variable, this will support the Challenger’s Character Hypothesis. We note that in our revised models we also entered the variables [Challenger’s strategic valence] and [Incumbent’s strategic valence] separately, in order to evaluate whether challengers’ positioning responds differently to their own strategic valence than to the strategic valence of the incumbent.

Table S2, which presents the parameter estimates for the model specifications described above, displays consistent support for the Challenger’s Character Hypothesis. For each model specification the coefficient estimate on the [Challenger’s character-based valence] variable is negative and statistically significant, indicating that, ceterus paribus, challengers moderate their ideological positions as their character-based valence improves—a pattern that is in line with our theoretical results. By contrast, the coefficient estimates on the [Incumbent’s character-based valence] variable are near zero and are not statistically significant. Note that these estimates are positive, indicating that as the incumbent’s character-based valence improves the challenging candidates tend to adopt more radical strategies, as our theoretical results suggest they should. However, the coefficient estimates on the [Challenger’s character-based valence] variable are of much greater magnitude than the estimates on the [Incumbent’s character-based valence] variable, which supports the Challenger’s Character Hypothesis, that challenging candidates adjust their policy strategies in response to their own character-based valence to a greater extent than they adjust to the character traits of the incumbent.

Finally, we emphasize an additional point about what our (non)finding, that congressional incumbents’ character-based valences do not exert statistically significant effects on challengers’ policy positioning, implies about challenger strategies. This finding does imply that, ceterus paribus, challengers’ policy strategies do not vary substantially in response to the incumbent’s character-based valence. However this pattern does not imply that the challenger’s desire that the incumbent deliver good government (in the event the incumbent is re-elected) is irrelevant to the incumbent’s policy strategy. For to the extent that our model captures real-world challengers’ motivations, the (missing) relationship between incumbents’ character-related valence and challenger positioning that we identify arises because when an incumbent displays bad character, then the challenger faces strategic cross-pressures: specifically, the challenger’s desire for good government creates incentives for her to moderate her policy strategy in order to

\textsuperscript{11} As long as $a_R \leq a_D$, the challenger’s strategy moderates more for an increase in her own character-based valence than that of the incumbent if $(-0.040 + 0.033a_R) < (-0.046 + 0.105a_D)$, i.e., if $a_D > 0.08$. 

unseat the poorly performing incumbent (a centripetal incentive), but at the same time the fact that voters observe the incumbent’s bad character improves the challenger’s electoral prospects, so that she has additional leeway to present noncentrist policies that are closer to her sincere beliefs (a centrifugal incentive). Our empirical analyses suggest that these centripetal and centrifugal incentives largely cancel each other out—but note that it is the fact that challengers intrinsically value the incumbent’s character-based valence that counteracts the strategic incentives related to the electoral effects of incumbent character. By contrast, our empirical findings suggest that because voters lack comparable levels of information about challengers’ character traits, high-character challengers do not derive significant electoral advantages from these traits and so they do not have the leeway to announce noncentrist policies—so that in the case of high-character challengers the centripetal incentive to moderate their policy strategies in order to win election and enjoy good government is the dominant motivation. Thus in our model, challengers’ tastes for the public good of good government affect their strategic reactions to both their own character-based valence and that of their opponent: It is this taste that pushes high-character challengers to moderate their policy positions, and it is this taste that deters challengers competing against low-character incumbents from shifting towards the more radical positions that they would otherwise present.
Table S1. Regression Estimates for Computer Simulation

A. Same character-salience parameter for both candidates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter estimates</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$s_D \in Unif[1,4]$</td>
<td>$s_D = 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$V_{SR} - V_{SD}$</td>
<td>0.088 (0.007)**</td>
<td>0.166 (0.0003)**</td>
<td></td>
</tr>
<tr>
<td>$s_D$</td>
<td>-0.135 (0.007)**</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.010 (0.021)</td>
<td>0.004 (0.001)**</td>
<td></td>
</tr>
<tr>
<td>$V_{CR} - V_{CD}$</td>
<td>-0.065 (0.015)**</td>
<td>-0.028 (0.001)**</td>
<td></td>
</tr>
<tr>
<td>$\alpha \ast (V_{CR} - V_{CD})$</td>
<td>0.097 (0.026)**</td>
<td>0.163 (0.001)**</td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.35</td>
<td>0.997</td>
<td></td>
</tr>
</tbody>
</table>

B. Candidate-specific character-salience parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter estimates</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$s_D \in Unif[1,4]$</td>
<td>$s_D = 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$V_{SR} - V_{SD}$</td>
<td>0.079 (0.007)**</td>
<td>0.165 (0.0004)**</td>
<td></td>
</tr>
<tr>
<td>$s_D$</td>
<td>-0.134 (0.007)**</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>$\alpha_R$</td>
<td>0.002 (0.020)</td>
<td>0.003 (0.001)**</td>
<td></td>
</tr>
<tr>
<td>$V_{CR}$</td>
<td>-0.040 (0.021)*</td>
<td>-0.029 (0.001)**</td>
<td></td>
</tr>
<tr>
<td>$\alpha_R \ast V_{CR}$</td>
<td>0.033 (0.034)</td>
<td>0.163 (0.002)**</td>
<td></td>
</tr>
<tr>
<td>$\alpha_D$</td>
<td>0.012 (0.022)</td>
<td>0.002 (0.001)</td>
<td></td>
</tr>
<tr>
<td>$V_{CD}$</td>
<td>0.046 (0.023)*</td>
<td>0.030 (0.001)**</td>
<td></td>
</tr>
<tr>
<td>$\alpha_D \ast V_{CD}$</td>
<td>-0.105 (0.038)**</td>
<td>-0.165 (0.002)**</td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.34</td>
<td>0.996</td>
<td></td>
</tr>
</tbody>
</table>

Notes. The dependent variable is the challenger’s strategy $s_R$. Standard errors are in parenthesis. The symbol (*) indicates statistical significance at the 0.05 level; (**) denotes significance at the 0.01 level. The definitions of the independent variables are given in the main text of the paper.
Table S2. Analysis of House Challengers’ Positions in the 2006 Elections (N=81), with Character-Based and Strategic Valence Separated By Candidate

<table>
<thead>
<tr>
<th></th>
<th>Baseline Model (1)</th>
<th>Primary Electorate Model (2)</th>
<th>Primary Electorate + District Partisanship Model (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Challenger’s character-based valence</strong></td>
<td>-.45** (.13)</td>
<td>-.49** (.12)</td>
<td>-.50** (.12)</td>
</tr>
<tr>
<td><strong>Incumbent’s character-based valence</strong></td>
<td>.04 (.17)</td>
<td>.12 (.16)</td>
<td>.20 (.22)</td>
</tr>
<tr>
<td><strong>Challenger’s strategic valence</strong></td>
<td>.22* (.10)</td>
<td>.22* (.10)</td>
<td>.24* (.10)</td>
</tr>
<tr>
<td><strong>Incumbent’s strategic valence</strong></td>
<td>-.20 (.23)</td>
<td>-.18 (.22)</td>
<td>-.20 (.22)</td>
</tr>
<tr>
<td><strong>Democratic incumbent</strong></td>
<td>.24 (.18)</td>
<td>.04 (.18)</td>
<td>.07 (.19)</td>
</tr>
<tr>
<td><strong>Incumbent distance from district</strong></td>
<td>-.15 (.11)</td>
<td>-.16 (.10)</td>
<td>-.15 (.11)</td>
</tr>
<tr>
<td><strong>Challenger’s partisans’ distance from district</strong></td>
<td></td>
<td>.75** (.23)</td>
<td>.62* (.31)</td>
</tr>
<tr>
<td><strong>District partisanship (relative to challenger)</strong></td>
<td></td>
<td></td>
<td>.17 (.27)</td>
</tr>
<tr>
<td><strong>Intercept</strong></td>
<td>1.56** (.33)</td>
<td>0.56 (.44)</td>
<td>0.66 (.47)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>.186</td>
<td>.281</td>
<td>.275</td>
</tr>
</tbody>
</table>

** p ≤ .01; * p ≤ .05, two-tailed tests.

Notes. In these analyses the dependent variable was the distance between the challenger’s ideological position and the mean ideological position of the voters in the district, measured along a 1–7 liberal-conservative scale. The definitions of the independent variables are given in the text of the paper “When Candidates Value Good Government.” The negative coefficient estimates on the [Challenger’s character-based valence] variable indicate that as the challenger’s character-based valence improves then, ceterus paribus, the challenger’s ideological position becomes more moderate relative to the district (i.e., the ideological distance between the challenger and the mean district voter declines).