Centrifugal Incentives

In Multicandidate Elections

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Previous versions of this paper were presented at the quarterly meeting of the Southern California Methodology Program, February 12, 1999, at the California Institute of Technology, the annual meeting of the Public Choice Society, March 12-14, 1999, in New Orleans, and the annual meeting of the American Mathematical Society, January 19-22, 2000, in Washington, DC.

Keywords: Conditional logit model, multicandidate election, Nash equilibrium, party identification, spatial model.
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Abstract

This paper analyzes factors that affect candidates’ position-taking incentives in multicandidate and multiparty elections. Following Cox (1990), we define centrifugal incentives as those that motivate vote-seeking candidates to take more extreme positions relative to the center of the voter distribution. For a multivariate vote model that includes a Left-Right policy component, a party identification component and an unmeasured term that renders the vote choice probabilistic, we present theoretical and computer simulation results that quantify candidates’ incentives to shift their policies away from the center in the direction of their partisan constituencies’ mean policy preferences. Centrifugal incentives are found to increase with (1) the salience of policies and party identification, (2) the size of the candidate field, (3) the size of a candidate’s partisan constituency, and (4) more extreme constituency policy preferences. Thus, ceteris paribus, candidates who represent large constituencies are motivated to present more extreme policies than are candidates who represent small ones.
Over the past five years numerous studies have appeared that analyze candidates’ and parties’ policy strategies in historical elections. These studies employ the approach pioneered by Erikson and Romero (1990) in their analysis of the 1988 American presidential election: namely, they estimate the parameters of a voting model from election survey data, and then compute the effects of alternative candidate/party policies upon their expected votes. Such studies thus combine the perspectives of behavioral research, which emphasizes the empirical study of voting behavior, and spatial modeling, which focuses on the policy strategies of vote-seeking candidates. The objective is to understand the policies that candidates and parties presented in historical elections, and (in some cases) to analyze the possibility of a candidate equilibrium -- a set of stable positions which vote-seeking candidates would present in order to win election. To date this research strategy has been extended outward from the United States to encompass elections in Canada (Alvarez, Nagler, and Willette 1999; Dow 2001), Britain (Alvarez, Nagler, and Bowler 2000), France (Adams and Merrill 2000), Germany (Schofield et al. 1998), Norway (Adams and Merrill 1999a, 1999b), the Netherlands (Schofield et al. 1998), and Israel (Schofield et al. 1998).

Although the studies cited above build upon the Erikson-Romero methodology, they differ from this study in two important respects. First, while Erikson and Romero focused on two-candidate competition all subsequent studies analyze multiparty/multi-candidate elections, i.e., elections involving at least three competitors. Second, Erikson and Romero demonstrated that two-candidate elections motivate candidates to present identical, centrist platforms, whereas subsequent studies conclude that in the historical multiparty contests under review the competing candidates and parties had diverse policy
incentives, in that some candidates could maximize votes by presenting centrist positions while others could do best by presenting noncentrist policies.

The multiparty studies cited above reach conflicting conclusions about how diverse the competing parties’/candidates’ policies would be in practice. Several of these studies conclude that in the historical election under review, all of the competing candidates could maximize votes by presenting similar, centrist policies, although these policies would not be identical (see Alvarez and Nagler 1995; Alvarez, Nagler, and Bowler 2000; Alvarez, Nagler, and Willette 1999; Dow 1997; Schofield, Sened, and Nixon 1998). However, studies of different historical elections find that candidates and parties had diverse policy incentives, with one or more competitors maximizing votes by presenting distinctly noncentrist positions (Adams and Merrill 1999a, 1999b, 2000; Schofield et al. 1998; Adams, 2001b, ch.6). These conflicting conclusions stem in part from variations across historical elections, including differences in the number of candidates and variations in the distributions of the electorates’ policy preferences – variables that spatial modelers typically consider in their analyses of candidate strategies (see, e.g., Cox 1990; Eaton and Lipsey 1975). However, other factors relate to voters’ decision-making processes, notably the importance voters attach to policies relative to unmeasured variables. In addition, there is massive empirical evidence that voters are influenced by measurable factors not directly related to the candidates’ policies in the current election, such as the party identification and sociodemographic characteristics of the voters and the charisma and competence of the competing candidates. These issues are central to the behavioral voting literature but have only recently received attention from spatial modelers (we review these studies below).
To date there exist no studies which systematically explore how all of the variables discussed above affect candidates’ and parties’ position-taking incentives in multicandidate elections. This is the issue we address in this paper. Specifically, we analyze quantitatively how changes in these variables affect vote-seeking candidates’ incentives to present centrist versus extreme policy positions. Following Cox (1990), we define centripetal incentives as those that motivate candidates and parties to present centrist policies, and centrifugal incentives as those that reward more extreme positions (we define the terms “centrist” and “extreme” more precisely below).

In analyzing policy strategies, we focus on the study of candidates’ and parties’ policies in Nash equilibrium, i.e., in situations where all competitors are configured so that no competitor can improve its expected vote by unilaterally modifying its policies. Party equilibria are important because they can be expected to be stable provided that the electoral context does not change, and several of the empirical studies listed above conduct their analyses in terms of candidate/party equilibria.

Although we explore the characteristics of candidate/party equilibria in vote-maximizing strategies, we recognize that party leaders and candidates care about more than simple vote-maximization. Both sets of actors plausibly weigh policy objectives (see, e.g., Calvert 1985; Wittman 1983), while party leaders in parliamentary democracies may anticipate postelection coalition negotiations (Austen-Smith and Banks 1988; Muller and Strom 1999; Schofield, Sened, and Nixon 1998; Strom 1990; Martin and Quinn, 2000). Nevertheless, votes obviously matter to parties and candidates, so that the electoral incentives we analyze may be important determinants of candidate and party behavior.

We develop a spatial model of unidimensional candidate competition, in which
voters are influenced by policies, party i.d., and a random component, and first present a theorem on the configurations of candidates’ vote-maximizing positions in situations where policies have low salience to the electorate. We then develop, for scenarios in which policies are more salient to voters, a combined theoretical and simulation analysis that expresses each candidate’s Nash equilibrium position as a function of the election parameters. Although our analyses can be extended to multidimensional spatial models – and/or to models incorporating abstention – we present our work in the context of a single (Left-Right) dimension in order to focus on how changes in election parameters affect candidates’ centrifugal (and centripetal) incentives.

Finally, we emphasize that two specifications of our voting model, that partisanship influences the vote and that rival parties’ partisans may display different spatial distributions, are treated as exogenous in the spatial model we investigate. Ideally, our analysis would illuminate the origins of voter partisanship and of differing partisan distributions, which we shall see are crucial to understanding candidate strategies. However these issues – which have preoccupied behaviorists for over 40 years (see, e.g., Campbell et al. 1960; Fiorina 1981; Converse 1969; Pierce 1992) – lie beyond the scope of this paper. We note that in treating these voter motivations as exogenous, we employ the same approach displayed in recent studies by Ansolabehere and Snyder (2000); Erikson and Romero (1990); Feld and Grofman (1991); Groseclose (1999); and Macdonald and Rabinowitz (1998), all of which analyze two-candidate elections. Thus we see our central contribution as extending the spatial analysis of measured nonpolicy voting influences from the two-candidate to the multicandidate setting.
A Simple Behavioral Voting Model

In the model let each voter $i$’s utility for each candidate $k$, $U_i(k)$, be a function of: the voter’s utility for $k$’s Left-Right position, specified by quadratic loss; $i$’s party identification $t_{ik}$, which is equal to 1 if the voter identifies with candidate $k$’s party and zero otherwise; and a random disturbance term $\varepsilon_{ik}$ (whose distribution is given below):

$$U_i(k) = -a(x_i - s_k)^2 + bt_{ik} + \varepsilon_{ik},$$

(1)

where $x_i$ and $s_k$ represent the positions of voter $i$ and candidate $k$ along the Left-Right dimension, and the parameters $a$ and $b$ represent the electoral salience of Left-Right ideology and partisanship, respectively. We specify that $a$ is positive and that $b$ is nonnegative. We adopt the convention of referring to candidates as “she” and voters as “he.”

Our specification, that the voter’s party identification influences his evaluation of the candidates independently of his ideological position, is supported by empirical voting research which finds, for instance, that left-wing Democratic partisans (in the U.S.) vote for Democratic candidates at higher rates than do left-wing independents, that right-wing Gaullist partisans (in France) support Gaullist candidates at higher rates than do right-wing partisans of the National Front, and so on (see Markus and Converse, 1979; Converse, 1969; Alvarez and Nagler, 1995; Pierce, 1995). This finding is consistent with the “Michigan model” of voting (Campbell et al., 1960), in which partisanship is
conceptualized as a long-term, effective orientation towards one’s preferred party – one
which grows out of such factors as early socialization experiences and positive evaluations
of the party’s past performance – and which is largely independent of the candidates’

Our vote-choice model is *conditional logit*, a model that has been used extensively
in empirical voting studies (see Endersby and Galatas 1998; Burden and Lacy 1999;
Adams and Merrill 1999a, 1999b, 2000a). Under this model the probability $P_{ik}(s,a)$ that
voter $i$ votes for candidate $k$ is

$$P_{ik}(s,a) = \frac{\exp\left[-a(x_i - s_k)^2 + bt_{ik}\right]}{\sum_j \exp\left[-a(x_i - s_j)^2 + bt_{ij}\right]},$$

(2)

where $s$ is the vector of all candidate positions.$^\text{iv}$

The voting model specified by equations 1-2 is similar to the one that Adams and
Merrill (1999a, 1999b, 2000) employ in their analysis of voting behavior and spatial
strategies in Norway and France, and is also similar to those employed by Endersby and
Galatas (1998) and Dow (1997) in their studies of British and French voting behavior,
respectively. While these studies incorporate additional nonpolicy variables relating to
voters’ sociodemographic characteristics (class, income, etc.), the results in Adams and
Merrill (2000) suggest that inclusion of such additional variables does not greatly change
the configuration of candidate equilibria, compared with equilibria obtained for the
specification presented in equations 1-2. Hence results obtained for this simple behavioral
voting specification should shed light on the equilibria that are likely to exist in more
complex models.
Parameters Relevant to the Election Context

With respect to the election context, we assume that the election involves $n \geq 3$ candidates, i.e. that this is a multicandidate election, and that a proportion $m_k$ of all voters identify with candidate $k$’s party (to be called partisans of candidate $k$ for short). A proportion, $m_0$, of voters are independent; their utilities have no partisan component. Thus

$$\sum_{k=0}^{n} m_k = 1.$$  

Suppose further that the partisans of the various candidates have spatial locations, which follow continuous distributions with means $\mu_k$ and probability density functions $f_k$. Without loss of generality, we specify that $\sum_{k=0}^{n} m_k \mu_k = 0$, i.e., the mean voter location $\mu_v$ is centered at zero along the Left-Right policy scale. We refer to the location $\mu_v = 0$ as the center of the voter distribution. Thus each partisan mean $\mu_k$ represents the policy distance (positive or negative) between the candidate’s partisans and the overall voter mean. We also assume that the mean location of independent voters $\mu_0$ is the same as the overall voter mean, i.e., $\mu_0 = 0$. Although this assumption apparently limits the generality of our model, a comparison of the overall voter means to the mean placements of independent voters in recent elections in France, Norway, and the United States, suggests that the assumption is approximately satisfied in all three electorates.
The theoretical results on candidates’ vote-maximizing strategies

The vote share of candidate $k$ is given by the expected value

$$EV_k(s,a) = \sum_i P_{ik}(s,a).$$

The partial derivative of $EV_k(s,a)$ with respect to $a$, evaluated at $a = 0$, achieves a maximum when $s_k$ assumes the value

$$s_k(0) = \frac{\sum_i P_{ik}(s,0)[1 - P_{ik}(s,0)]x_i}{\sum_j P_{jk}(s,0)[1 - P_{jk}(s,0)]} = \sum_i w_{ik}(0)x_i,$$

where $s$ is the vector of candidate positions and $P_{ik}(s,0)$ denotes the probability that voter $i$ chooses candidate $k$ when the policy parameter, $a$, is zero, i.e., when candidate choice is determined entirely by voter-specific nonpolicy attributes and the random components of voter utility, and $w_{ik}(0)$ is given by

$$w_{ik} = \frac{P_{ik}(s,0)[1 - P_{ik}(s,0)]}{\sum_j P_{jk}(s,0)[1 - P_{jk}(s,0)]}.$$ 

In other words, as a policy component is introduced into the model, the candidate’s expected share of the vote increases most rapidly for this strategy. It turns out that the values $s_k(0)$, although seemingly only tangentially related to policy considerations, form the primary basis for determining optimal (Nash equilibrium) strategies when policy motivations are present.

This relationship between optimum strategies when policy salience is low and the corresponding strategies when it is more realistic occurs for the following reason. As we will see from computer simulations below, the configuration of equilibrium strategies
varies continuously as the policy parameter, $a$, varies over the range suggested by analysis of historical elections. Furthermore, as $a \to 0$, each candidate $k$’s equilibrium strategy $s_k(a) \to s_k(0)$, where the latter is given by eqn. (3) above. Accordingly, the configuration of the $s_k(0)$ are a good guide to the configurations when $a$ is substantially above zero.

Note that the $s_k(0)$ do not constitute an equilibrium, but rather each $s_k(0)$ will be shown to be a component of the equilibrium position of the $k^{th}$ candidate when the policy salience coefficient is not zero. We will provide an explicit algebraic formula for $s_k(0)$ and an approximate formula for the residual component of $s_k(a)$.

The values $s_k(0)$ are given in Theorem 1 by a formula that involves exactly five determinants: (1) the mean ideal location $\mu_k$ of the candidate’s partisans, (2) the proportion $m_k$ of all voters who identify with the candidate’s party, (3) the proportion $m_0$ of independent voters, (4) $n$, the number of candidates, and (5) the value of the partisan salience parameter $b$. As we will see, this result permits us -- under rather general conditions -- to approximate Nash equilibrium strategies in terms of a small number of parameters and to delineate the nature of the effect of each determinant on the optimal strategy.

**Theorem 1.** If the mean $\mu_0$ of the independent voters is the same as the overall mean $\mu_V$, then

$$s_k(0) = c_k \mu_k$$ (5)

where
\[ c_k = \frac{(n-2)(e^b-1)m_k}{(n-2)(e^b-1)m_k + \left(e^b + n - 2\right) + m_0\left[e^b + n - 1\right] \left[\frac{n-1}{n^2}\right] - \left(e^b + n - 2\right)} . \]  \hspace{1cm} (6)

**Proof**: See Appendix A.

We next determine a relationship similar to that suggested by Theorem 1 for situations where voters attach significant weight to policies. To determine the needed Nash equilibria, we applied – to simulated elections -- an iterative algorithm (see Merrill and Adams 2001) that can compute equilibrium configurations (if any) to any degree of accuracy.

**Illustrative Examples**

First, for illustration, we consider a 7-point scale from –3 to 3 and five candidates, whose partisans are normally distributed about the mean positions -2, -1, 0, 1, and 2, respectively, with a standard deviation of 0.75 for each partisan distribution. Two distributions of the proportions of party identifiers will be illustrated (for these illustrations, there are no independents). Under distribution A, all parties are of equal size, i.e., \( m_i = 0.2, i = 1, \ldots, 5 \). For distribution B, the vector of proportions are \( (0.1, 0.3, 0.2, 0.3, 0.1) \), representing a pattern that occurs frequently in real electorates (e.g., France, Norway) in which a center-left and a center-right party are dominant in size with extreme parties being smaller.

Using the algorithm for Nash equilibria, we computed candidates’ equilibrium locations for the partisan distributions A and B described above, while varying the values
of the policy salience coefficient \( a \). For these computations the party i.d. salience parameter \( b \) was set at 2.0, about the value estimated for historical elections in Norway and France, and the value of \( a \) was set at \( a=0.01,0.02,\ldots,0.20 \). These values for \( a \) and \( b \) encompass the range of values for these parameters suggested by empirical studies of voting in France (Adams and Merrill 2000; Fleury and Lewis-Beck 1993); Britain (Endersby and Galatas 1998); Norway (Adams and Merrill 1999a, 1999b); and the U.S. (Alvarez and Nagler 1995). For all values of \( a \) that we investigated we obtained equilibrium configurations, which are plotted as functions of \( a \) in Figures 1A-1B. Note that, consistent with Theorem 1, as \( a \) approaches zero each candidate’s equilibrium position approaches \( s_k(0) \). Furthermore, it is apparent that as \( a \) increases above zero, the pattern of the equilibrium configuration remains similar but simply expands, roughly linearly.

Simulation Analysis

To test the relationships illustrated above, we next applied the algorithm to randomly generated election scenarios, in which we varied the number of candidates \( n \) (varied between 3 and 7), the means \( \mu_k \) of the partisan groups (each chosen randomly from a uniform distribution between –2.5 and 2.5), the policy salience coefficient \( a \) (from a uniform distribution between 0 and 0.20), the partisan salience coefficient \( b \) (from a uniform distribution between 1.0 and 3.0), and the proportions \( m_k \) of partisans that fall in each party group as well as the proportion \( m_0 \) of independent voters. For five hundred
elections (100 each with three candidates, four candidates,..., seven candidates) were simulated, with parameters chosen as described above, generating a total of 2500 simulated candidates.

The first finding to report is that equilibria were found to exist for all 500 of the randomly generated election scenarios. This result contrasts with the situation for simple deterministic multicandidate models, where in general equilibria do not even exist (Eaton and Lipsey 1975). It is consistent, however, with equilibrium analyses of multiparty elections in Israel (Schofield, Sened, and Nixon 1998), Norway (Adams and Merrill 1999a, 1999b), and France (Adams and Merrill 2000), which are based on probabilistic voting models. Given that the voting parameters used in the simulations were chosen to reflect the range of estimates reported in empirical voting studies, these simulation results – in combination with the empirical studies cited above – strongly suggest that equilibria in vote-maximizing strategies will exist in realworld multicandidate elections.

An Approximation Formula for Nash Equilibrium Strategies

The algorithm allows us to compute equilibria precisely, but does not reveal the dependence of the locations at equilibria on the various parameters. For that reason, we seek to express $s_k(a)$, the equilibrium position of the $k^{th}$ party, directly by an algebraic formula. We succeed in obtaining an approximation to such a formula, based on computer simulation.

With respect to the effect of the policy salience coefficient $a$ upon candidates’ equilibrium locations, the simulated data and Figures 1A-1B suggest that the residual $s_k(a) - s_k(0)$ is roughly proportional to $a$ and increases rapidly at first as $s_k(0)$ increases,
then more slowly, suggesting proportionality to $s_k(0)^{1/2}$. Scaling considerations suggest that the residuals are also proportional to the overall variance, $\sigma^2_V$, of the voter distribution.\textsuperscript{ix} Empirically, we find that $s_k(a) - s_k(0)$ increases approximately proportional to $\ln(n-1)$, the natural logarithm of the number of candidates minus one. Applying multiple regression to data from Monte Carlo simulation, we are thus led to the following approximation formula:

$$s_k(a) \approx s_k(0) \pm 0.5a\sigma^2_V\sqrt{s_k(0)}\ln(n-1)$$

(7)

where the coefficient 0.5 is estimated from the regression, the plus sign is used if $\mu_k$ is positive, and the minus sign if it is negative. For fixed $n$, the configuration of equilibrium positions $s_k(a), k=1,\ldots,n$, is an expansion of the configuration of the $s_k(0), k=1,\ldots,n$; the degree of expansion depends primarily on policy salience. Figure 2 plots the exact equilibrium positions obtained by the algorithm against the approximate equilibrium positions predicted by the right hand side of eqn. (7). The R-squared for this regression is 0.982, the mean absolute value of the error on a 1-7 scale is 0.023, and 97 percent of the prediction errors are less than 0.1, i.e., in most cases the formula given in eqn. (7) is a quite accurate approximation for $s_k(a)$.
Thus, by the approximation eqn. (7), for a given value of $a$ each candidate $k$’s equilibrium position $s_k(a)$ becomes more extreme for more extreme values of $s_k(0)$. Hence the factors that cause $s_k(0)$ to become more/less extreme will exert similar effects for higher degrees of policy voting. In general, conclusions about the centrifugal (or centripetal) effects of the variables $n$, $m_k$, $m_0$, $b$, $\mu_k$, $s_k(0)$ extend, at least approximately, to scenarios in which $a$ significantly exceeds zero.

Implications for Candidates’ Vote-maximizing Positions

Centrifugal Incentives for Candidate Strategies

We now consider the question, what does the theorem and the approximation formula (7) imply about policy strategies for vote-seeking candidates? We note that for the special case $b = 0$, i.e. when partisanship does not influence the vote, it follows from eqns. (6) and (7) that $c_k = 0$ and hence candidate optima will be agglomerated at or very near the center, a result consistent with work by Lin, Enelow, and Dorussen (1999), for multicandidate probabilistic spatial voting (see also de Palma et al. 1989; Adams 1999a). If, however, $b=0$ but $a$ is not near zero – a likely scenario in a realistic model with no partisanship term – an equilibrium agglomerated at the center may be far from unique, leading to great uncertainty in predicting outcomes (see Merrill and Adams 2001).

Matters are quite different for the more general case where voters display partisan biases (i.e., $b > 0$). In this situation it is easily verified from eqn. (6) that if $m_k > 0$, then $0 < c_k < 1$, and hence by eqn. (5) that $s_k(0)$ lies between the mean voter ideal point
\( \mu_v = 0 \) and the mean position \( \mu_k \) of the party’s partisans. As has been illustrated before in specific cases (Adams and Merrill 1999a, 1999b, 2000; Adams, 2001b, chapter 6), we conclude that when voters display partisan biases, each candidate’s vote-maximizing position is shifted away from the center, in the direction of the mean position of her partisans. Although this result may seem intuitively compelling, a pure spatial model, even with a probabilistic component does not predict it.

Why, specifically, are vote-seeking candidates motivated to shift away from the center in the direction of their partisans? The reason is that the marginal change in a candidate’s probabilities of attracting her own partisans’ votes via policy appeals is higher than is the marginal change in her probabilities of attracting rival candidates’ partisans. To understand why this is true, note that the properties of the conditional logit (CL) probability function imply that the weight \( w_{ik} \) that a candidate \( k \) attaches to a voter \( i \)’s policy preference is maximal when the probability that \( i \) votes for \( k \) is 0.5.xv In multicandidate competition the highest of a voter’s vote probabilities must be the one nearest to 0.5, and hence the voter is most marginal with respect to the candidate he is most likely to support. Because, ceteris paribus, partisan voters are more likely to vote for their party’s candidate than for a rival party’s candidate, candidates attach greater weight to their own partisans’ beliefs than to the beliefs of rival parties’ partisans. Therefore these candidates’ optima will be shaded in the direction of their partisans’ preferences.

The Quantitative Effects of Model Parameters

We next consider the question: What factors determine how far candidates’ optima diverge from the mean voter position \( \mu_v = 0 \)? First, note that the approximation formula
(7) suggests that when voters exhibit partisan biases (i.e. \( b > 0 \)), the candidates tend to shift farther away from the center with increases in the policy salience parameter \( a \) and increases in the population variance parameter \( \sigma_v^2 \). Hence the more voters emphasize policies and the more dispersed the electorate’s policy preferences, the more extreme the candidates’ optimal positions. The latter result on population variance is consistent with previous work by Cox (1990), and makes intuitive sense: the more dispersed the distribution of voters’ ideal points, the more dispersed we should expect vote-seeking candidates’ positions to be at equilibrium.

With respect to the partisan mean \( \mu_k \), it follows from eqn. (5) that \( s_k(0) \) – and hence \( s_k(a) \) – increases with \( \mu_k \). Hence the more extreme the position of a candidate’s partisans, the more extreme the candidate’s optimal position.xvii

The remaining four variables relevant to candidate equilibria – \( m_k \), \( m_0 \), \( n \), and \( b \) – also influence the candidates’ optima through their effect upon \( s_k(0) \) (see equations 5 and 7). Theorem 1 establishes that the values \( s_k(0) \) are obtained by shrinking \( \mu_k \), the mean position of candidate \( k \)'s partisans, by a factor \( c_k \), so that values of \( c_k \) near 1.0 indicate that \( s_k(0) \) is near \( \mu_k \), while values of \( c_k \) near zero indicate that \( s_k(0) \) is near the overall voter mean \( \mu_v = 0 \). Hence we consider the effect of each of the variables \( m_k \), \( m_0 \), \( n \), and \( b \) upon \( c_k \).

First, the shrinkage coefficient \( c_k \) given by eqn. (6) increases with \( m_k \), the proportion of candidate \( k \)'s partisans.xviii Thus the larger the candidate’s partisan constituency, the more extreme the candidate’s optimal position. In particular, small parties that have an extreme constituency may face a quandary. The optimal strategy of
such a party, given our model, may be near the center – perhaps nearer the center than that of larger, more moderate parties – but such a strategy would seriously undermine its credibility.

Second, the mathematical expression given on the RHS of eqn. (6) implies there is no clear-cut relationship between increases in the partisan salience parameter \( b \) and the degree to which candidates present extreme as opposed to centrist policies. Numerical calculation of \( c_k \), however, for a range of values of \( b \) (see Table 1A) shows that \( c_k \) increases with \( b \) as long as \( b \) does not exceed about 2 or 3 (depending on the values of \( m_0 \)) but declines thereafter. \textsuperscript{xix} Since empirical studies suggest that the values of \( b \) in historical elections usually fall below this cutoff point, this suggests that candidate optima typically become more extreme as partisan salience increases. \textsuperscript{xx} 

With respect to the number \( n \) of candidates, for any \( b > 0 \), \( c_k \) increases in value as \( n \) increases. \textsuperscript{xxi} Thus the more candidates there are the more extreme the candidate’s optimal position. We note that this conclusion is consistent with theoretical results by Cox (1990) on spatial competition for deterministic policy voting, who finds that for a variety of electoral systems, incentives for policy dispersion increase with the number of competitors.

Finally, simple algebra shows that the coefficient of \( m_0 \) (the proportion of independent voters) in the expression for \( c_k \) is positive if \( b > 0 \) and \( n \geq 3 \). That is to say, \textit{ceteris paribus, optimal strategies become more centrist as the proportion of independents increases.} This is not unexpected because, as noted above, previous results by Lin, Enelow, and Dorussen (1999) have established that if all voters are independents and if the
policy component $a$ is sufficiently small, then the overall voter mean is a Nash equilibrium for all candidates.

Table 1B shows how $c_k$ varies with the number of candidates, $n$, and the size of candidate $k$'s partisan constituency, $m_k$, when $b = 2$ and the proportion of independents is set at $m_0 = 0.3$, about the value observed in many Western European voting publics (see Converse 1969; Pierce 1995, chap.3; Sinnott 1998, fig. 1). These results show, as expected, that $c_k$ increases with the number of candidates and with the size of the candidate's partisan constituency.

Note that in Table 1B as in Table 1A, the computed values of $c_k$ typically fall below 0.5, so that when the policy salience coefficient $a$ is near zero the candidate’s vote-maximizing position is located nearer to the overall voter mean than it is to the mean of the candidate’s partisans. Finally, as can be seen from the proof of Theorem 1, the values of $s_k(0)$ depend on the distributions of the partisan constituencies only through their means, $\mu_k$.

We emphasize two caveats about our use of the terms “centrist” and “extreme” in the above discussion. First, these terms are defined relative to the mean position of the electorate rather than against some independent ideological standard, so that for instance if the electorate is overwhelmingly Marxist in orientation, then candidates that reflect the electorate’s views are considered centrist relative to the voter distribution (see, e.g., Cox 1990, 213). The second, related, point is that when we speak of candidates becoming “more extreme” in response to changes in the election variables $n$, $m_k$, etc., this denotes
only that the parties are shifting away from the voter mean, not that their positions are extreme in some absolute sense.

**An Empirical Illustration**

We illustrate the precision of the approximation formula (7) with data drawn from the 1988 French Presidential Election Study (Pierce 1996). Because this election featured five major candidates (Lajoinie, Mitterrand, Barre, Chirac, and Le Pen), it provides a good test of the expected divergence of optimal strategies in the presence of a large candidate field. As well as partisan self-identifications, the study obtained both self-placements and candidate placements from respondents on a Left-Right scale -- a dimension that taps into economic policy. Because the French election is a presidential elections between candidates rather than between parties (as would be the case in a proportional representation system), partisan identification is not the same as candidate preference and hence represents in part an independent factor affecting vote choice.xxi

Using the respondents’ self-placements on the Left-Right dimension (N = 748) for voter ideal points, their mean candidate placements for the candidates’ actual positions, their reported vote choices, and their partisan self-identifications, we estimated the policy and the partisan parameters $a$ and $b$ in eqn. (1) from maximum likelihood in a conditional logit model. These are reported in the bottom two rows of Table 2. We also computed the mean positions and the standard deviations of each candidate’s partisan constituency as well as the proportions of partisans in each constituency (see Table 2, columns 2-5). Note that the standard deviations of the partisan constituencies are in the vicinity of 0.5 to 1.0, the range we used in the simulations. Using the algorithm, we computed both the
candidates’ exact equilibrium positions and their approximate equilibrium positions as calculated from eqn. (7). These values are reported in columns 6 and 7 of Table 2.

As expected from our simulations, we find that the French candidates’ equilibrium positions and their approximate equilibrium locations obtained from formula (7) are quite similar to each other, with no candidate’s actual computed optimum differing from the position predicted from the formula by more than 0.04 units along the 1-7 Left-Right scale. We also note that each candidate’s optimum is similar to but less extreme than the mean position of his partisan constituency, results consistent with our theoretical analysis.xxiii Note that the optimal policy locations of Lajoinie and Le Pen, the candidates with small constituencies, are substantially closer to the center than are the policy optima for Mitterrand and Chirac, who represent larger constituencies. This is to be expected in light of our theoretical conclusion that larger partisan constituencies exercise greater centrifugal effects on candidate strategies. The more extreme actual positions of Lajoinie and Le Pen suggest, however, that other incentives such as policy motivations exert further centrifugal effect on candidate positions.

Figures 3A-3C show how the candidates’ equilibrium positions vary with changes in $a$, $b$, and $\mu_k$. Consistent with our previous simulation results, we located equilibria for all values of the parameters that we investigated. This suggests that an equilibrium is to be expected for the 1988 French presidential election, given realistic voting parameters.
Consistent with our simulation results, we find that candidate strategies become more extreme as policy voting increases, with the optima expanding roughly linearly with increases in $a$. We find, also consistent with our theoretical and simulation analyses and for $b \leq 2$, that increases in the electoral salience of partisanship exercise moderate centrifugal effects on candidate strategies, with the candidates adopting more extreme positions as voters become increasingly partisan. These results also support our conclusion that vote-seeking candidates have incentives to shift in the direction of their partisans’ beliefs. Figure 3C shows how equilibrium configurations vary as a function of the mean positions $\mu_k$ of the candidates’ constituencies. For this exercise the positions of all partisan voters were multiplied by a scale factor, $\beta$, which we varied between 0.5 and 2.5. As expected, Figure 3C shows that the candidate optima become more extreme with increases in $\beta$. This supports our theoretical conclusion that the more extreme the positions of parties’ partisans, the more extreme the parties’ vote-maximizing positions.

Discussion

While spatial modelers have produced many findings on two-candidate elections, they have developed fewer results relevant to the multiparty/multicandidate elections that are the norm outside the United States, particularly in the context of choice according to the multivariate voting model of behavioral research. We have tried to provide a systematic account of the strategic logic of multiparty competition when voters choose according to a simple behavioral voting model (conditional logit) that incorporates
policies, partisanship, and random components – the latter representing influences that are not measured in standard election surveys.

We conclude that in multicandidate elections, vote-seeking candidates are motivated to shift away from the center in the direction of their partisans’ policy beliefs, and that increases in the following variables motivate candidates to take more extreme positions relative to the center of the voter distribution: the electoral salience of policies, the electoral salience of partisanship, dispersion of the electorate’s policy preferences, the size of the candidate field, and, for each candidate, the size of the candidate’s partisan constituency and the extremity of the positions of the candidate’s partisans. However, an increase in the number of independent voters provides a centripetal incentive, which motivates candidates to present more centrist positions. We found that the candidates’ equilibrium positions could be predicted with great accuracy from a formula involving seven parameters \((a, b, m_k, m_0, \mu_k, n, \sigma^2_V)\), with the mean error of the predictions in our simulations falling below .025 policy units for each candidate along the 1-7 policy scale.

Voters’ nonpolicy motivations are important for understanding the strategic logic of multicandidate spatial competition. Intuitively, one might expect that even if measured nonpolicy variables such as partisanship, class, and race influence the vote, they are irrelevant to candidate positioning since they cannot be easily manipulated in the course of a campaign. Our analysis extends the findings in Adams and Merrill (1999a, b, 2000; see also Adams 2001a b) that this intuition is mistaken: voter partisanship strongly affects the nature of candidates’ optimal strategies, an effect we must take into account. What are new are our quantitative conclusions about the strategic implications of voter partisanship. For while our results on the centrifugal incentives associated with increases in the number
of parties and the electoral salience of policies have been anticipated in the spatial modeling literature\textsuperscript{xxvi}, our conclusions on the centrifugal effects associated with increases in the partisan-related variables $b$, $m_k$, and $\mu_k$ have not.\textsuperscript{xxvii}

Finally, we find that parties/candidates maximize votes by presenting policies similar to \textit{but less extreme than} their partisans’ beliefs. Thus, in realworld multiparty elections, vote-seeking parties and candidates should generally pursue moderate policies. This implies in turn that smaller, more extreme parties such as the French Communists and the National Front, the German Greens, and the Norwegian Progress Party are forgoing electoral gains in pursuit of alternative goals, such as policy objectives. Such parties cannot attempt to maximize their support by moderating their policies without forgoing both their credibility and the policy goals of their activists.\textsuperscript{xxviii}

In closing, we emphasize again that a complete understanding of parties’ or candidates’ policy proposals in historical elections must move beyond an exclusive focus on vote-maximization, to encompass the party elites’ policy motivations, their calculations about postelection governing coalitions, the electoral costs involved (in terms of credibility) in changing the party’s policy programme, and the electoral laws in effect in the historical election of interest (see Cox 1990, 1997; Dow, 2001). This large number of complicating factors suggests that we should treat our conclusions on candidate strategies with caution. The above considerations notwithstanding most analysts believe that votes are very important to politicians, so that the electoral incentives we have analyzed may be a major factor driving political elites’ policy decisions. Hence we believe that our conclusions about centrifugal incentives in multiparty elections can shed light on the strategic logic of party and candidate competition – and on the observed policy behavior of
parties and candidates in historical multiparty elections.
Appendix. Proof of Theorem 1

**Theorem 1.** Assuming that the mean, $\mu_0$, of the independent voters is the same as the overall mean, $\mu_r$, which we take to be 0, the quantities $s_k(0)$ defined in eqn. (7) are given by:

$$s_k(0) = c_k\mu_k$$

where the $c_k$ are defined by

$$c_k = \frac{(n-2)(e^b-1)m_k}{(n-2)(e^b-1)m_k + (e^b+n-2) + m_0\left[(e^b+n-1)^2\left(\frac{n-1}{n^2}\right) - (e^b+n-2)\right]}.$$

**Proof:** Suppose that the partisans of the various parties have spatial locations, which follow continuous distributions with means $\mu_k$ and probability density functions $f_k$. We fix a party $k$, $k = 1, \ldots, n$. Because $P_{ik}(s,0)$ is the same for all partisans of the same party, we may denote by $P_{kj} = P_{ik}(s,0)$ the probability that a voter $i$ who is a partisan of party $j$ votes for party $k$. Thus, if voter $i$ is a partisan of party $k$:

$$P_{kk} = \frac{e^b}{e^b+n-1}.$$
If, instead, voter $i$ is a partisan of another party $j$, then:

$$P_{bj} = \frac{1}{e^b + n - 1}. $$

Similarly, if voter $i$ is an independent, then:

$$P_{k0} = \frac{1}{n}. $$

Recall that we have assumed, without loss of generality, that $\sum_j m_j \mu_j = \mu_0 = 0$. By assumption, $\mu_0 = 0$ so that $\sum_{j=0}^n m_j \mu_j = 0$. Because all terms other than $x_i$ in the expression for $s_{k0}$ are independent of the voter locations, asymptotically, we have:

$$s_k(0) = \frac{P_{kk} [1 - P_{kk}] m_k \int_{-\infty}^{\infty} x f_k(x) dx + \sum_{j \neq k, 0} P_{kj} [1 - P_{kj}] m_j \int_{-\infty}^{\infty} x f_j(x) dx + \int_{-\infty}^{\infty} x f_0(x) dx}{P_{kk} [1 - P_{kk}] m_k + \sum_{j \neq k, 0} P_{kj} [1 - P_{kj}] m_j + \int_{-\infty}^{\infty} x f_0(x) dx}$$

$$= \frac{P_{kk} [1 - P_{kk}] m_k \mu_k + \sum_{j \neq k, 0} P_{kj} [1 - P_{kj}] m_j \mu_j + \int_{-\infty}^{\infty} x f_0(x) dx}{P_{kk} [1 - P_{kk}] m_k + \sum_{j \neq k, 0} P_{kj} [1 - P_{kj}] m_j + \int_{-\infty}^{\infty} x f_0(x) dx}$$

26
\[
\frac{e^b(n-1)m_k \mu_k + e^b + (n-2) \sum_{j \neq k, 0} m_j \mu_j + \frac{1}{n} \left(1 - \frac{1}{n}\right)m_0 \mu_0}{e^b(n-1) \left(\frac{n-1}{e^b + n - 1}\right)^2 m_k + e^b + (n-2) \sum_{j \neq k, 0} m_j + \frac{1}{n} \left(1 - \frac{1}{n}\right)m_0}
\]

\[
e^b(n-1)m_k \mu_k + [e^b + (n-2)] \sum_{j \neq k, 0} m_j + 0
= \frac{e^b(n-1)m_k \mu_k + [e^b + (n-2)] \sum_{j \neq k, 0} m_j + 0}{e^b(n-1) + [e^b + (n-2)] \sum_{j \neq k, 0} m_j + \left[e^b + n - 1\right]^2 \left\lfloor \frac{n-1}{n^2} \right\rfloor m_0},
\]

because we have assumed that \( \mu_0 \), the mean of the distribution of independent ideal points, is zero. In turn, our expression is

\[
\frac{[e^b(n-1) - (e^b + n - 2)] m_k \mu_k + [e^b + (n-2)] \sum_{j \neq k, 0} m_j \mu_j}{[e^b(n-1) - (e^b + n - 2)] m_k + [e^b + (n-2)] \sum_{j \neq k, 0} m_j + (e^b + n - 1)^2 \left(\frac{n-1}{n^2}\right) m_0}
\]

\[
= \frac{(n-2)(e^b-1)m_k \mu_k}{(n-2)(e^b-1)m_k + (e^b + n - 2)(1 - m_0) + (e^b + n - 1)^2 \left(\frac{n-1}{n^2}\right) m_0}
\]

\[
= \frac{(n-2)(e^b-1)m_k \mu_k}{(n-2)(e^b-1)m_k + (e^b + n - 2) + m_0 \left[(e^b + n - 1)^2 \left(\frac{n-1}{n^2}\right) - (e^b + n - 2)\right]}
\]

27
\[ = c_k \mu_k, \]

where

\[
c_k = \frac{(n-2)(e^b - 1)m_k}{(n-2)(e^b - 1)m_k + (e^b + n - 2) + m_0 \left[ (e^b + n - 1)^2 \left( \frac{n-1}{n^2} \right) - (e^b + n - 2) \right]}. \]

This completes the proof.
References


Groseclose, Timothy (1999) ‘Character, Charisma, and Candidate Location: Downsian Models When One Candidate Has a Valence Advantage,’ Unpublished manuscript.


Table 1. Values of the Factor $c_k$, as a Function of Model Parameters

TABLE 1A. Values of $c_k$, for $n = 5$ and $m_k = 0.3$

<table>
<thead>
<tr>
<th>Value of the policy salience coefficient $b$</th>
<th>0</th>
<th>.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0=0$</td>
<td>0.112</td>
<td>.213</td>
<td>.295</td>
<td>.356</td>
<td>.399</td>
<td>.427</td>
<td>.445</td>
<td></td>
</tr>
<tr>
<td>$m_0=0.1$</td>
<td>0.111</td>
<td>.209</td>
<td>.285</td>
<td>.335</td>
<td>.361</td>
<td>.364</td>
<td>.346</td>
<td></td>
</tr>
<tr>
<td>$m_0=0.2$</td>
<td>0.110</td>
<td>.205</td>
<td>.274</td>
<td>.316</td>
<td>.329</td>
<td>.317</td>
<td>.284</td>
<td></td>
</tr>
<tr>
<td>$m_0=0.3$</td>
<td>0.109</td>
<td>.201</td>
<td>.265</td>
<td>.299</td>
<td>.303</td>
<td>.281</td>
<td>.240</td>
<td></td>
</tr>
<tr>
<td>$m_0=0.4$</td>
<td>0.108</td>
<td>.197</td>
<td>.256</td>
<td>.283</td>
<td>.280</td>
<td>.252</td>
<td>.208</td>
<td></td>
</tr>
<tr>
<td>$m_0=0.5$</td>
<td>0.107</td>
<td>.193</td>
<td>.248</td>
<td>.270</td>
<td>.261</td>
<td>.228</td>
<td>.183</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 1B. Values of $c_k$, for $b = 2$ and $m_0 = 0.30$

<table>
<thead>
<tr>
<th>Proportion of party identifiers $m_k$</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n=3$</td>
<td>0</td>
<td>.052</td>
<td>.098</td>
<td>.140</td>
<td>.179</td>
<td>.214</td>
</tr>
<tr>
<td>$n=4$</td>
<td>0</td>
<td>.092</td>
<td>.168</td>
<td>.233</td>
<td>.288</td>
<td>.336</td>
</tr>
<tr>
<td>$n=5$</td>
<td>0</td>
<td>.124</td>
<td>.221</td>
<td>.299</td>
<td>.362</td>
<td>.415</td>
</tr>
<tr>
<td>$n=7$</td>
<td>0</td>
<td>.173</td>
<td>.295</td>
<td>.386</td>
<td>.456</td>
<td>.512</td>
</tr>
<tr>
<td>$n=10$</td>
<td>0</td>
<td>.221</td>
<td>.362</td>
<td>.460</td>
<td>.532</td>
<td>.587</td>
</tr>
</tbody>
</table>

Note: $n$ = the number of parties, $b$ = the partisan salience parameter, $m_0$ = the proportion of independents, and $m_k$ = the proportion of partisans of party $k$. 

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<table>
<thead>
<tr>
<th>Candidate</th>
<th>Proportion of partisans</th>
<th>Mean placement of candidate</th>
<th>Mean location of partisans</th>
<th>Standard deviations of partisans</th>
<th>Equilibrium positions (algorithm)</th>
<th>Approximate positions (formula 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lajoinie</td>
<td>6.6%</td>
<td>1.90</td>
<td>2.24</td>
<td>0.83</td>
<td>3.69</td>
<td>3.70</td>
</tr>
<tr>
<td>Mitterrand</td>
<td>39.2%</td>
<td>3.09</td>
<td>3.08</td>
<td>0.69</td>
<td>3.52</td>
<td>3.48</td>
</tr>
<tr>
<td>Barre</td>
<td>13.5%</td>
<td>4.81</td>
<td>4.85</td>
<td>0.68</td>
<td>4.24</td>
<td>4.23</td>
</tr>
<tr>
<td>Chirac</td>
<td>18.0%</td>
<td>5.55</td>
<td>5.36</td>
<td>0.71</td>
<td>4.44</td>
<td>4.43</td>
</tr>
<tr>
<td>Le Pen</td>
<td>4.5%</td>
<td>6.57</td>
<td>6.03</td>
<td>1.17</td>
<td>4.23</td>
<td>4.21</td>
</tr>
<tr>
<td>Other</td>
<td>18.2%</td>
<td>--</td>
<td>4.02</td>
<td>1.01</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

\[ a \text{ (Policy salience)} = 0.204 \]
\[ b \text{ (Partisan salience)} = 2.14 \]

Log-likelihood: -622.6

Note: *Partisans* are defined as identifiers of the parties represented by the candidates indicated. The category *other* includes identifiers of other parties as well as those respondents who did not identify with any party.
Figure 1. Nash equilibrium positions versus the policy-salience parameter, $a$ (simulated data)

A. Five parties of equal size
B. Five parties of size (0.1, 0.3, 0.2, 0.3, 0.1)
**Figure 2.** Scatterplot of Exact Nash Equilibrium Positions versus Approximate Positions Predicted by Equation (7)
Figure 3. Equilibrium positions of candidates in the 1988 French presidential election, as functions of each of three parameters

A. Equilibrium as a function of the policy-salience parameter
Figure 3 (continued)

B. Equilibrium as a function of the partisan-salience parameter
Figure 3 (continued)

C. Equilibrium as a function of the spread of the voter-distribution
Many researchers have studied parties’ one-step optima, i.e. the positions that maximize parties’ expected votes when all rival parties locate at their actual positions. Results reported by Adams and Merrill (1999b, Table 2) and Merrill and Adams (2001, Table 3) suggest that for the voting model we study here, parties’ one-step optima are quite similar to their equilibrium positions, i.e., vote-maximizing parties can be expected to move very rapidly toward their equilibrium positions.

To avoid unnecessary repetition, below we discuss spatial strategies in the context of elections involving candidates, although all of our conclusions apply equally to candidate- and to party-centered contests.

Since the publication of The American Voter (Campbell et al. 1960), party identification has been singled out by behavioral researchers as uniquely important.

Under conditional logit assumptions, for each candidate \( k \) the random component \( \varepsilon_{ik} \) in eqn. (1) is generated independently from a type I extreme value random distribution. Two alternatives to conditional logit are multinomial probit and the Generalized Extreme Value model, each of which allow for nonzero correlations between the error terms \( \varepsilon_i \) (see Alvarez and Nagler 1998). We do not explore these models because here we are not concerned with the effects of such correlations upon candidate strategies. For an analysis of this issue see Adams 1999b.

In the 1988 French Presidential Election Survey (see below), the mean self-placement of independents was 4.02 (on a 1-7 scale) compared to a mean of 3.97 for all respondents. In the 1988 American NES, mean placement for independents (those reporting a “3” on the 0-6 scale) was 4.16, compared to 4.37 for all respondents. In the 1989 Norwegian Election Study, the mean for independents was 5.85 (on a 1-10 scale) and that for all respondents, 5.53.

Note that \( P_{ik}(s, 0) \) is independent of \( s \).

This notation means that 10 percent of the voters fall in the party whose supporters are centered at -2; 30 percent in the party whose supporters are centered at -1, etc.

The proportion of nonpartisans, \( m_{oi} \), was generated from a uniform distribution between 0 and 0.5, which is roughly the observed range for the proportions of nonpartisans in a number of historical elections. The
partisan proportions $m_k$ were generated from a uniform distribution -- normalized so that together with $m_o$, they summed to 1 -- and under the restriction that no candidate’s party would have more than ten times the partisans of any other. The latter was done to eliminate minor parties which would be hard to compare with real parties, since national election surveys typically contain only a few data points on partisans of such parties. The standard deviation of the positions of the partisans of each party, and for independent voters, was chosen uniformly from 0.5 to 1.0, roughly the range of deviations suggested by survey data, as shown below.

ix We define $\sigma_V^2$ as $\sigma_V^2 = \sum m_k (\mu_k^2 + \sigma_k^2)$, where $\sigma_k^2$ is the variance of partisans of party $k$. The quantity $a \times \sigma_V^2$ is invariant under changes in scale.

x The values of R-squared and prediction errors reported here were obtained by calibrating formula (7) from one simulated data set and then applying it to a fresh set of simulated data.

xi This is because the second term on the RHS of eqn. (7) is $0.5a\sigma_V^2 \sqrt{s_k(0)} \ln(n - 1)$ and by inspection this term is invariably positive.

xii One of the variables on the RHS of eqn. (7), $n$, affects the party’s approximate equilibrium location $s_k(a)$ both through its affect on $s_{00}$ and also through its inclusion in the second expression on the RHS of eqn. (7). Thus increases in $n$ exercise centrifugal effects in two ways: by making $s_{00}$ more extreme, and by increasing the value of $0.5a\sigma_V^2 \sqrt{s_{00}} \ln(n - 1)$.

xiii In addition, note that while we are concerned here with multicandidate competition, it follows from eqn. (6) that for two-candidate elections ($n=2$), $c_k = 0$ even if $b > 0$ and hence by eqn. (5) two-candidate equilibria will be agglomerated, a conclusion consistent with results reported by Erikson and Romero (1990), for two-candidate probabilistic spatial voting with measured nonpolicy influences.

xiv This is because the denominator in eqn. (6) is equal to the numerator plus two positive terms.

xv This is also true for the multinomial probit (MNP) probability function, in the general case where the correlations between the error terms associated with voters’ candidate utilities are set to zero, and the error terms have equal variances.
This is because the second expression on the RHS of equation 7, \( 0.5a\sigma_f^2\sqrt{s_{k0}}\ln(n-1) \), increases with \( a \) and with \( \sigma_f^2 \).

Note that this result depends only on the means of the voter distribution functions, not on other aspects of the distributions such as the degree of dispersion of each party's identifiers.

To see this, it suffices to show that \( 1/c_k \) decreases as \( m_k \) increases. Because, by eqn. (6),

\[
\frac{1}{c_k} = 1 + C \left( \frac{1 - m_0}{m_k} \right) + \frac{D m_0}{m_k},
\]

where \( C \) and \( D \) are positive constants, it is clear that if \( m_0 \) is constant, then \( 1/c_k \) decreases as \( m_k \) increases. It can be shown that this relation continues to hold if \( m_0 \) decreases by no more than \( m_k \) increases.

Note that for \( m_0 = 0 \), \( \frac{1}{c_k} = 1 + C \left[ (e^b - 1) + (n-1) \right] \), where \( C \) is a positive constant. The latter clearly decreases as \( b \) increases.

While behavioral researchers disagree sharply about the electoral impact of partisanship (see Fleury and Lewis-Beck 1993; Converse and Pierce 1993), the empirical estimates of \( b \) noted above typically vary between approximately \( b=1 \) and \( b=2.5 \), suggesting that, if \( a \) remains constant, increases in \( b \) will usually be associated with increases in \( c_k \).

It suffices to show that \( 1/c_k \) decreases with \( n \). But

\[
\frac{1}{c_k} = 1 + \frac{1}{(e^b - 1)m_k} \left[ \frac{(1-m_0)e^b}{n-2} + (1-m_0) + m_0 \left( \frac{e^b + n - 1}{n^2} \right) \frac{(n-1)}{(n-2)} \right].
\]

The first two terms in brackets decrease with \( n \) or are constant. The final term in brackets is of the form \( m_0 \left( \frac{n + \varepsilon}{n^2} \right) \left( \frac{n-1}{n-2} \right) \), where \( \varepsilon = e^b - 1 > 0 \). But \( \frac{n + \varepsilon}{n} = 1 + \frac{\varepsilon}{n} \) decreases with \( n \) and so does \( \frac{n-1}{n-2} = 1 + \frac{1}{n-2} \), which...
completes the argument.

In addition, as noted above $n$ also enters equation 7 through the second expression on the RHS,

\[ 0.5a\sigma_y^2 \sqrt{\ln(n-1)} . \]

However, note that like the first expression on the RHS of equation 7, this second expression also increases with $n$, which supports our conclusion that candidate optima become more extreme as the number of competitors increases.

xxii 19% of the French partisans voted for a candidate from a different party.

xxiii Extension of the model to incorporate directional motivations suggested by Rabinowitz and Macdonald (1989) imply that the optimal strategies of the three major candidates are nearer their constituencies (see Merrill and Adams 2000).

xxiv In these calculations all election parameters were held constant at their observed values as reported in Table 2, except for the single parameter of interest.

xxv Because $a\sigma_y^2$ is invariant under changes in scale, the policy-salience parameter, $a$, was varied inversely with the square of the scale factor, $\beta$.

xxvi As noted above, our conclusion on the centrifugal effects associated with increases in the number of parties has been anticipated by Cox (1990; see also Eaton and Lipsey 1975), and our conclusion on the centrifugal effects associated with increases in the salience of policies has been anticipated by Lin, Enelow, and Dorussen (1999; see also Adams 1999b; de Palma et al. 1989; de Palma, Hong, and Thisse 1990). In addition, while we are unaware of previous work on multicandidate elections that explores the effects of increasing the dispersion of the electorate’s policy preferences, it strikes us as common sense that an increase in voter dispersion creates centrifugal incentives for candidates.

xxvii Adams and Merrill (1999b; see also Adams 1998) present illustrative arguments that parties have electoral incentives to appeal to their partisans on policy grounds, but these authors do not present theoretical results to this effect, nor do they systematically explore how this motivation varies with the proportion of partisans ($m_k$), nor with the mean position of the partisans’ policy preferences ($\mu_k$).

xxviii Consistent with this analysis, elsewhere we report empirical applications to election survey data from France and Norway that suggest that the larger parties (and their candidates) located near their computed
vote-maximizing positions while the smaller parties did not (see Adams and Merrill 1999b, 2000).