What are the Effects of Entry of New Extremist Parties on the Policy Platforms of Mainstream Parties? *

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ABSTRACT

We examine the consequences to policy-seeking, center-left and center-right parties under proportional representation following entry by extremist parties either at one or both ends of a unidimensional political spectrum. Assuming a symmetric, unimodal voter distribution, we show that the entry of a single extremist party on either the left or right drives both mainstream parties in the direction opposite to the extremist party. We argue that this three-party scenario is the most empirically relevant case in contemporary European politics. We also extend results of Casamatta and De Donder that project moderation of mainstream parties at equilibrium for PR elections with two extremist parties -- one on each end -- to a symmetric, unimodal voter distribution. In a setting not considered by these authors, we show that this moderating effect on the mainstream parties for a symmetric voter distribution is reversed if the voter distribution is sufficiently bimodal.
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I. Introduction

Adams and Merrill (2006) trace the consequences of the entry into unidimensional political competition by a centrist party in a two party situation with single-seat competition under plurality where there is a center-right party and a center-left party, both of which are policy-seeking, i.e., entry that changes a two-party game into a three-party game. While Duvergerian ideas of the dynamics of plurality based competition (see Duverger, 1954; Palfrey, 1984; Fey, 1997) might suggest that the center-right and the center-left party would operate to squeeze out the center party by each moving further to the center, Adams and Merrill demonstrate the counterintuitive result that, if the major parties are both policy seeking parties, they can be expected to propose policies that are much more divergent than what we should observe without the third party.

While new parties of many stripes may enter the political process, a common phenomenon today, especially in European polities, is the appearance of an extreme party that substantially cuts into the support of more centrist parties. Most commonly this extremist party is a populist party on the right (see Widfeldt, 2014; Eatwell, 2017; Kitschelt and McGann, 2017; Mudde, 2017; Abou-Chadi and Krause, 2018). We model this situation as a three-party game, with a center-right and a center-left party initially, followed by the entry of an extremist party – thus converting a two-party game into a three-party game. Such a scenario can be thought of as a natural complement to the Adams-Merrill (2006) set-up.

We first highlight the consequences of the entry of a single extremist party at one end of the political spectrum, since we believe this scenario has considerable empirical relevance, not just in PR settings which are the focus of this paper, like the Netherlands, but also in plurality settings, such as the UK, and in two-round elections, such as in France. But sometimes we have extremist parties entering the political space from both ends of the political spectrum -- although usually in recent decades it is the right-wing party that it is the greatest threat to the center. This possibility leads us to consider the
four-party case, with a center right and a center left party at the beginning followed by entry of more extreme parties from both sides of the political spectrum. When there are two new extremist parties, one at each end of the spectrum, we extend findings of Casamatta and De Donder (2005) from log-concave voter ideal densities to any symmetric unimodal densities. Then we provide a new theorem that extends their work further, by examining what happens when the underlying voter distribution is bimodal.

We see our modeling as very relevant to events in the real world in terms of the decline in the vote share of mainstream parties throughout much of Europe (including the UK) and the response of established parties to new and more extreme ones that begin to garner substantial voter support. Our set-up allows us to consider both the empirically common case where it is a new (or renewed) right-wing party that is the greatest threat, and the case where the threat to the center comes from both ends of the spectrum.

1. Empirical Evidence about the Appearance of Extremist Parties and the Response to them of Mainstream Parties

Throughout Europe there are challenges to mainstream parties by more extremist parties. For example, since 2006, the Dutch Party for Freedom (PVV), which has taken anti-immigration views similar to those of the late Pim Fortuyn, has had parliamentary representation. After winning 24 seats in 2010 and joining, then withdrawing from a coalition government with mainstream parties on the center right, this party dropped to 15 seats and out of government. In the UK, UKIP, a Eurosceptic and nationalist yet populist political party has gone from nowhere to being the largest British party in terms of members of the European Parliament and, although winning only one MP in the House of Commons, received the third highest vote share in the last parliamentary election. Moreover, with a hard-exit Brexit ever more likely as of this writing (January 2019), the importance of UKIP was enhanced because of the role they played in successfully lobbying for Brexit and an anticipation (until internal divisions within UKIP surfaced) that they would make inroads into areas of traditional Labour support as well in areas of traditional Conservative Party support. Similarly, in France, there is the potential for the Le Penistes, who offer a strong nationalist anti-immigration stance, to consistently

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1 Internal divisions were exposed within the Labour party after the Brexit vote and the travails of Labour leader Jeremy Corbyn for behavior interpreted as endorsing terrorists.
become its second most important party. In these countries, the major challenge to mainstream parties from extremist parties seems to be coming from only one end of the political spectrum.\(^2\)

Previous empirical work has looked at the reaction of center parties to the emergence of extremist parties on their flanks, but the conclusions have been mixed. Analyzing cross-national data, however, to test competing expectations is not that straightforward. On the one hand, there can be long run time trends that operate largely independently of the structure of the party system. Alonso and Fonseca (2012) note that, in their study of 18 western European countries over the period 1975-2005, mainstream parties of the left and right have trended in an anti-immigration direction regardless of whether or not extreme right parties have been present. For example, in Austria, Germany, the Netherlands, and Sweden, the mainstream Left moved in an anti-immigration direction before the emergence of significant far right parties. On the other hand, some empirical research that focuses on immigration issues (see Bale et al., 2010; among others) suggests that the mainstream right has responded to the rise of radical right parties with a harder line on immigration. But both Alonso and Fonseca (2012) and Akkerman (2015) conclude that the degree of mainstream accommodation reported in this previous research is overstated.

2. *Theoretical Expectations for Mainstream Response*

The absence of clear empirical findings suggests the desirability of modeling the incentive structure of the center parties more formally, so as to develop better understanding of the factors that affect whether centrist party response to new extreme parties is one of accommodation or one of rejection. The modeling that is closest to our own is the ground-breaking work of Casamatta and De Donder (2005), who obtain explicit equilibria for the cases of two or four parties, under the assumptions that the voter distribution is log concave, but do not consider the case of three parties, when a

\(^2\) This seems true even when there is a new green party attracting some support. In virtually all countries facing real populist party challenges the greens do not do as well as the populists.
single extreme party is present. While their work also includes the plurality case, which we mention briefly below (see footnote 11 in section II.1), as well as results for PR systems, we focus on extending this work in the PR context. We contribute in four ways: (1) most importantly for the potential empirical relevance of our modeling results, we obtain new results for the case of three parties when only one extreme party is present, (2) we prove the equilibrium results for any unimodal, symmetric voter distribution, (3) given two extremist parties, one on each flank, we demonstrate divergence of the mainstream parties at equilibrium for a sufficiently bimodal (polarized), symmetric distribution, and (4) we consider two different assumptions about how coalitions might form.

As in Casamatta and De Donder (2005) and Adams and Merrill (2006), we assume that the motivations of the two mainstream parties are policy seeking and that party utility is a function of distance to the parliamentary mean. In this unidimensional setting, when parties are policy seeking and the voter distribution is symmetric and unimodal, we show that entry of only one extremist party moves both mainstream parties in the opposite direction from that party. For symmetric, unimodal voter distributions, the entry of two extremist parties again, by contrast, drives both the center-right and the center-left party further toward the center. In one sense, this result is the opposite of what happens when a centrist party joins the competition under the Adams-Merrill (2006) model. In another sense, however, the findings here for the effect of extreme parties and the findings by Adams and Merrill (2006) for the effect of a centrist party are similar. In

3 We thank an anonymous reviewer for calling our attention to this previous work.

4 A unimodal distribution need not be log-concave [Ibragimov 1956]. Because log-concavity is assumed by Casamatta and De Donder (2005), our results extend the applicability of the equilibrium results to the slightly broader class of unimodal voter distributions -- by assuming a condition that is easier to conceptualize and apply in practice than log-concavity.
each case, the mainstream parties are driven away from the newly entering party or parties.⁵

We assume that, in the short run, the mainstream parties are free to move but that the position of the extreme party is fixed, which we believe realistically reflects the behavior of extreme parties, especially in their early years.⁶ For example, Jean-Marie Le Pen maintained a quite consistent stance on immigration and related topics for decades; it was only when his daughter Marine inherited the party that there was any real ideological movement—the renunciation of anything directly smacking of racism and anti-Semitism that accompanied her purge of her father from the party. But even she maintained the party’s anti-immigration stance. More generally, Adams et al. (2006), in a study of eight Western European democracies from 1976-1998, find that niche or ideologically-based parties typically did not respond to shifts in public opinion (or were punished at the polls if they did), while mainstream parties did respond to public opinion shifts, without paying electoral penalties.

However, any tendency of mainstream parties to move away from entering extreme parties can disappear or be reversed under alternative conditions, including bimodality of the voter distribution and/or discounting by the mainstream parties of the relevance of the extreme parties to the competition to join a governing coalition. The political climate in some countries may be such that particular parties, normally extremist parties, are not deemed “worthy” of participating in coalitions. Thus, as we suggest later, the policy locations of those parties may be less influential on the platform choices of mainstream parties than are the choices made by other mainstream parties. Moreover, the appearance of extreme parties may proceed hand in hand with increasing bimodality of the voter distribution, as a synergy develops between rejection of establishment norms in

⁵ Adams and Merrill (2014) observe this same consequence of extreme-party entrance in the context of office-seeking mainstream parties and uncertainty about the location of the voter distribution. They note that their result counters the intuition that a policy-oriented extreme party might enter the race for the purpose of drawing the proximate mainstream party toward its preferred policy.

⁶ Numerical computations suggest that our results are not greatly affected if the extreme party is allowed to move.
the electorate and appeals by extremist parties. Such effects can provide the conditions under which mainstream parties would respond to extreme party entry by moving to positions that are more, rather than less, extreme. We may summarize our findings in the following three statements:

1) Given either one or two extreme parties, if the voter distribution is symmetric and unimodal and mainstream parties account for all parties in assessing the expected policy outcome (i.e., place it at the parliamentary mean), then they should move away from the extreme party(s).

2) If the voter distribution is symmetric and unimodal but mainstream parties disregard extreme parties in determining the expected policy outcome (i.e., place it at the mainstream party mean), then the mainstream parties should stay put.

3) If the voter distribution is symmetric and sufficiently bimodal, however, we show that the mainstream parties should move to more extreme stands when extreme parties enter on their flanks, i.e., right of center mainstream parties should shift further right and left of center mainstream parties should shift further left.

II. Formalizing the Model

Employing a one-dimensional spatial model, represented by a line centered at 0, we investigate the changes in the equilibrium ideological locations of two mainstream

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Faced with an extreme party on its right flank, a center-right party might move decisively to the right with the intention of eliminating the extreme party by co-opting its appeal (Meguid 2005, 2008). Such a strategy is more likely if the underlying distribution of attitudes on the populist dimension is bimodal, and when support for the populist position is coming from both traditionally center-right and traditionally center-left voters. It appears that the Conservative Party in Britain followed that strategy in the aftermath of the Brexit vote by adopting key elements of the UKIP manifesto.
parties, one a center-left party, one a center-right party, for scenarios in which new and more extreme parties are added to the party system. Initially there are two parties: a mainstream left party located (with notation chosen in anticipation of the introduction of further parties) at a point $x_2$ between -1 and 0, and a mainstream right party, located at position $x_3$ between 0 and +1, so that overall, $-1 \leq x_2 \leq 0 \leq x_3 \leq 1$. In the three-party model, an extreme party enters on the right, at some fixed location, $x_4 \geq 1.0$. In the four-party model, an additional extreme party enters on the left, at the fixed location $x_i \leq -1.0$. We assume that voters choose the party closest to their position. As noted earlier, in our theoretical work, we focus on the case where the electoral system is a proportional one, though our modeling framework has potential application to the plurality case as well.

1. Policy-seeking Motivations

In this paper we assume that the parties are policy-seeking, i.e., each party seeks to optimize its utility for the expected outcome. Policy-seeking assumptions were pioneered by Wittman (1973, 1977) and have been pursued by Calvert (1985), Londregan and Romer (1993), Groseclose (2001), and Adams, Merrill, and Grofman (2005), among many others, including Casamatta and De Donder (2005). We assume throughout that seat assignments under proportional representation (PR) faithfully reflect the parties' vote shares, i.e., seat shares are exactly equivalent to vote shares. Furthermore, we assume that each party's utility for a policy position is based on linear loss, i.e., party $j$'s utility for position $x$ is given by $-|q_j - x|$, where $q_j$ is party $j$'s preferred (ideal) position. Finally, we assume that parties faithfully maintain the same policy intentions before and after election and government formation.

Each party's vote share is only one means toward the end of achieving a policy outcome. Because vote shares and participation in a government/governing coalition is

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8 Analysis of a three-party model with the extreme party on the left is similar; the details are omitted.
not known in advance, the policy outcome is probabilistic. For the initial models discussed in this section, all parties are assumed to be potential members of a governing coalition. We consider two alternative interpretations of policy seeing goals and show that, under certain assumptions, these two interpretations lead to the same mathematical formulation. Thus, whichever interpretation is more realistic, or whether some intermediate assumption would be most appropriate, the analyses and the conclusions would be the same.

Our first interpretation supposes that policy is a compromise among the proposals of all parties making up a parliament, weighted by their seat shares, without regard to whether a party is in government or opposition. Merrill and Adams (2007) comment that this parliamentary-mean model is perhaps most relevant to consensual democracies -- such as Switzerland, Belgium, and the Netherlands -- in which opposition parties' prerogatives provide a degree of policy influence that may approach that of the governing parties.9 Furthermore, they cite empirical support by Warwick (2001), in analyses of ten democracies, that the parliamentary mean is a significant predictor of government policy declarations.

Accordingly, given $N$ parties whose vote shares are $p_i, i = 1, \ldots, N$, we assume that party $j$ seeks to minimize the distance between its ideal location and the weighted mean of the actual locations of all parties, where the weights are the vote shares of the respective parties. This weighted mean, denoted by $\bar{x}$, where $\bar{x} = \sum_{i=1}^{N} p_i x_i$, is referred to as the parliamentary mean. Equivalently, party $j$ seeks to maximize its utility $U_j$ for the parliamentary mean, where this utility is the negative of the distance between its ideal point and the parliamentary mean, i.e.,

$$U_j = -|q_j - \bar{x}|$$

where $q_j, j = 1, \ldots, N$ denote the ideal points (preferred policy positions) of the parties.10

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9 As indicated above, Casamatta and De Donder (2005) obtain their results assuming a parliamentary-mean model.

10 Nash equilibria that are determined below for the parliamentary mean model are the same for linear and quadratic loss functions.
Alternatively, as Merrill and Adams (2007) note, we may assume that following the election one of the parties becomes primarily responsible for forming policy and that, a priori, the identity of this party is probabilistic, but related to the electoral strength of the parties. Under this dominant-party model in its pure form, the party awarded the opportunity to form a government (the formateur) might fully implement its preferred policies. Warwick (2001), in empirical analyses, suggests that the formateur does indeed derive policy advantages. Accordingly, instead of minimizing the distance to a parliamentary mean, participants might suppose that a single party would be expected to determine policy and that utility would depend on a lottery over the competing parties with weights proportional to the seat, and hence vote shares, i.e., \( U_j' = -\sum_i p_i |q_j - x_i| \).

Thus the dominant-party model could reflect a PR system in which policy is expected to be set primarily by a single party, but in which the identity of this party is not known in advance.\(^\text{11}\) If it is the case that the ideal points \( q_j \) are more extreme than all party

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\(^{11}\) Diermeier and Merlo (2004) report support for the proportionality hypothesis when no party receives a majority of the seats. There is evidence, furthermore that, taking into account relative importance, ministries are distributed approximately proportional to governing parties’ seat shares (Gamson, 1961; Browne and Franklin, 1973; Browne and Frendreis, 1980). Another interpretation of the dominant-party model involves an electoral system in which voting is by plurality in a winner-take-all context. If we assume that the probability that the \( i \)th party wins (control of government) is proportional to its vote share \( p_i \) (see section II.4 and the Appendix for a variant of this assumption), then utility is again a lottery over the competing parties, i.e., \( U_j = -\sum_i p_i |q_j - x_i| \). Thus, as before, if the party ideal points \( q_j \) are more extreme than all party positions, then \( U_j = -|q_j - \bar{x}| \) as in equation (1) above.
positions\textsuperscript{12} (i.e., if both $q_3 \geq x_4$ and $q_4 \geq x_4$, and similarly, $q_2 \leq x_1$ and $q_1 \leq x_1$ when one or both extreme parties are in the model), then this definition of $U_j'$ is equivalent to that of $U_j$ given by equation (1) above for the parliamentary mean model.\textsuperscript{13}

Since the formula for party utility for the outcome is the same for both the parliamentary-mean and the dominant-party models, provided the ideal points $q_j$ are more extreme than all party positions, this formula is also the same for an intermediate model that combines the aspects of both of these approaches (as long as the party ideal points are more extreme than all party positions).\textsuperscript{14} Such an intermediate model, in which policy is neither determined entirely by a single party nor arises as a complete compromise between all parties involved, appears more realistic than either pure model. With this robustness check in mind we proceed.

\textbf{2. The Effects on Mainstream Parties of Entry of New, Extreme Parties at Fixed Locations, for a Normal Distribution of Voter Points}

\textsuperscript{12} In the analysis later in the paper, we note that relaxing the assumption that party ideal points are more extreme than all party positions reduces the effects of an extreme party but does not qualitatively affect the conclusions (see sections II.2 and II.3).

\textsuperscript{13} The equivalency of $U_j'$ and $U_j$ follows because, under the conditions given,

$$U_3' = -\sum p_i |q_3 - x_i| = \sum p_i (x_i - q_3) = \sum p_i x_i - q_3 \sum p_i = \bar{x} - q_3 = -|q_3 - \bar{x}| = U_3,$$

and a similar calculation holds for $U_j'$, $j = 1, 2, \text{and 4}$.

\textsuperscript{14} This follows if party utilities for an intermediate model are defined as $\alpha U_j + (1 - \alpha)U_j'$, for then party utility under the intermediate model is given by

$$\alpha |q_j - \bar{x}| + (1 - \alpha) |q_j - \bar{x}| = |q_j - \bar{x}|,$$

as in equation (1).
We illustrate, for a normal voter distribution, the dynamic effects of successive entry of extreme parties (at fixed locations) upon the Nash equilibrium\(^{15}\) strategies of the mainstream parties. We assume that the voter distribution has mean 0 and standard deviation 0.5.

We begin with only two (mainstream) parties -- a center-left party and a center-right party. If, say, the center-right party moves right, it faces a trade-off because its position \(x_3\) improves but its vote share \(p_3\) decreases, so its contribution \(p_3x_3\) to the parliamentary mean can either increase or decrease, depending on the trade-off\(^{16}\).

For the parliamentary mean model, under the assumption that the ideal points of the mainstream parties straddle \(\bar{x}\), i.e., \(q_2 \leq \bar{x} \leq q_3\), Table 1 and Figure 1 indicate Nash equilibria for the mainstream parties as additional parties are added dynamically under the assumptions of normality. In the 2-party scenario, the optimal (Nash equilibrium) positions for the mainstream parties (±0.627) are about 1.25 standard deviations from the mean voter position. When, in addition to two mainstream parties, an extreme party enters on the right (at the fixed position, +1.0), the mainstream party on the right moderates its position to 0.477, a little less than one standard deviation from the mean.

This moderation can be intuitively explained because moderation draws vote share away from the center-left party (to the center-right party) as well as transfers vote

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\(^{15}\) A Nash equilibrium set of strategies is a configuration of party positions such that no party can unilaterally improve its utility by moving its position. Existence theorems for Nash equilibria are derived in Proposition 4 below.

\(^{16}\) Should the center-left party locate at the center point 0, the center-right party can enhance its utility by moving right. So the median is not a Nash equilibrium position for the parties (as it would be for office-seeking parties under a Downsian model). Note that Calvert (1985) obtains convergence to the median under policy-seeking assumptions that are quite different from the parliamentary-mean model (see details in the Appendix).
share from the center-right party (to the extreme right party) -- both of which move the parliamentary mean toward the center-right party's ideal point. These effects are partly counter-balanced by the fact that the center-right's more moderate position per se moves the parliamentary mean leftward. These effects suggest that, given a specific position for the center-left party, the center-right party achieves an interior equilibrium between the center point and the party's ideal point, as illustrated for the parliamentary-mean model by the solid arrow in the example in Figure 2. That the net impact of all these considerations favors moderation at equilibrium for the center-right party is explained below.

The mainstream left party, however, moves further to the left, in this case to location -0.777 (a little over 1.5 standard deviations from the mean). Thus, following the entry of an extreme party on the right, both mainstream parties move to the left, away from the extreme party. Roughly speaking, the center-left party, having lost vote share to the center-right party, attempts to make up its loss in utility by gaining desired policy position by moving left; the details are discussed below. Similarly, following entry of an extreme party on the left (but none on the right), both mainstream parties move right, again away from the extreme party.

Moving on to the four-party model, we note that adding an additional extreme left party to the 3-party scenario obviates the need of the mainstream left party to move left and both mainstream parties draw in toward the center to symmetric locations at \( \pm 0.427 \), each located less than a standard deviation from the mean (see Figure 1). Again, following the entry of extreme parties, each mainstream party moves away from its most proximate extreme party.

In the dominant party model, the Nash equilibrium results are the same as above for the parliamentary mean model, as long as we make the more stringent assumption that \( q_2 \leq x_1 \) and \( q_3 \geq x_4 \). Suppose, instead, that in the dominant party model the ideal locations of the mainstream parties are less extreme than the locations of the extreme parties (but still more extreme than their 2-party equilibrium locations). Then as extreme parties enter, the equilibrium locations of the mainstream parties move in the same
directions as for the parliamentary mean model, but these movements are less pronounced, as indicated in Table 1.\textsuperscript{17}

3. General Results for the Movements of the Mainstream Parties in Response to the Entry of Extreme Parties and Nash Equilibria

In this section we show that, in general, the response of mainstream parties to the entry of extreme party(s) is similar to that described above for a normal voter distribution and tabulated in Table 1.

As detailed above, voters are posited to choose the party closest to their position. We assume henceforth that voter ideal points have a distribution with mean equal to 0, specified by the density $f$ and the cumulative distribution function $F$ and that $f$ is differentiable. More specifically, we assume that the outcome of an election is specified by the parliamentary mean (although, as we have indicated above, other assumptions lead to the same party utility functions under certain conditions). Denote by $m_{jk}$ the midpoint between party positions $x_j$ and $x_k$, so that $m_{12} = (x_1 + x_2) / 2$, etc. Assuming proximity voting, in a 3-party scenario with locations $x_2$, $x_3$, and $x_4$, the vote shares of the parties are given by:

\begin{align*}
p_2 &= F(m_{23}) \\
p_3 &= F(m_{34}) - F(m_{23}) \\
p_4 &= 1 - F(m_{34}) ;
\end{align*}

Similarly, in a 4-party scenario with locations $x_1$, $x_2$, $x_3$, and $x_4$, the vote shares are given by:

\textsuperscript{17}This occurs because, under the dominant-party model, a mainstream party values the proximate extreme party less as a proxy than under the parliamentary-mean model. For the three-party scenario, Figure 2 illustrates the trade-offs as well as the equilibrium value for the center-right party under the dominant-party model, somewhat to the right of that under the parliamentary mean model (an effect that is similar in the three-party and four-party models).
\[ p_1 = F(m_{12}), \quad p_2 = F(m_{23}) - F(m_{12}), \quad p_3 = F(m_{34}) - F(m_{23}), \quad p_4 = 1 - F(m_{34}). \] (2b)

We assume, for now, that the two extreme parties, if present, are fixed at \( x_1 = -1 \) and \( x_4 = +1 \), respectively, and that these are the ideal points of these extreme parties (i.e., \( q_1 = x_1 \) and \( q_4 = x_4 \)).

Our first proposition shows that in the base model with only two parties, a Nash equilibrium exists and is unique, and we specify the values of the equilibrium locations for each party.

**PROPOSITION 1 (Nash equilibrium in a two-party model):** In a model consisting of only the two mainstream parties, assume that these parties are policy seeking based on the parliamentary mean, and that \( q_2 \leq \bar{x} \) and \( q_3 \geq \bar{x} \). Assume further that the voter density, \( f \), is symmetric and \( f(0) > 0 \). Then a unique Nash equilibrium occurs for \( x_2 = -1/(2f(0)) \) and \( x_3 = +1/(2f(0)) \).

**PROOF:** Casamatta and De Donder (2005)\(^{18}\) show that for a symmetric voter density, log concavity of \( F(x) \) (i.e., \( f(x)/F(x) \) is decreasing with \( x \)) is sufficient for a Nash equilibrium at \( x_2 = -1/(2f(0)) \) and \( x_3 = +1/(2f(0)) \). In the Appendix, we show that symmetry alone is sufficient for this result to hold.

**Definition:** A symmetric probability density \( f \) is **unimodal** if \( f(x) \) declines (or does not increase) as \( x \) recedes from the median on either side of that median.\(^{19}\)

**LEMMA 1:** Assume that the two mainstream parties are policy seeking based on the parliamentary mean, and that \( q_2 \leq \bar{x} \) and \( q_3 \geq \bar{x} \). If there is one extreme party \( x_4 \) on

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\(^{18}\) See also Ortuno-Ortin (1997) for related results.

\(^{19}\) To be precise, a symmetric distribution with median (mean) \( \tilde{m} \) is unimodal if when \( \tilde{m} \leq x_1 < x_2 \), then \( f(x_1) \geq f(x_2) \) and when \( x_1 \leq x_2 \leq \tilde{m} \), then \( f(x_1) \leq f(x_2) \).
the right (the three-party model), then the rate of change of utility of each mainstream party with respect to that party's location is given by

\[
\frac{\partial U_3}{\partial x_3} = \int_{m_{23}}^{m_{43}} f(x) dx - \left[ f(m_{23})(x_3 - m_{23}) + f(m_{34})(m_{34} - x_3) \right].
\] (3a)

\[
\frac{\partial U_3}{\partial x_2} = -\int_{m_{23}}^{m_{43}} f(x) dx + f(m_{23})(m_{23} - x_2)
\]

If there are two extreme parties \(x_1\) and \(x_4\), the first on the left and the latter on the right (the four-party model), then the rate of change of utility of each mainstream party with respect to that party's location is given by

\[
\frac{\partial U_3}{\partial x_3} = \int_{m_{12}}^{m_{43}} f(x) dx - \left[ f(m_{12})(x_3 - m_{12}) + f(m_{23})(m_{23} - x_3) \right].
\] (3b)

\[
\frac{\partial U_3}{\partial x_2} = -\int_{m_{12}}^{m_{43}} f(x) dx + \left[ f(m_{12})(x_2 - m_{12}) + f(m_{23})(m_{23} - x_2) \right]
\]

Suppose further that the voter density, \(f\), is symmetric and unimodal. If \(\frac{\partial U_3}{\partial x_3}\) is continuous, then, for either the 3-party or 4-party model and for any fixed value of \(x_2\), \(U_3\), reaches a maximum at some point \(x_3'\) on the interval from 0 to \(x_4\) and this point satisfies the recursive equation

\[
x_3 = \frac{p_3 + m_{23}f(m_{23}) - m_{34}f(m_{34})}{f(m_{23}) - f(m_{34})}.
\] (4)

Similar recursive equations for maximum points hold under similar conditions for \(x_2\) and \(x_3\) in the 3-party and 4-party scenarios.

**Proof.** See the Appendix.

Thus for example, in the 3-party scenario (see Figure 3),

\[
\frac{\partial U_3}{\partial x_3} = -A + B,
\] (5a)

where

\[
A = -\int_{m_{23}}^{x_3} f(x) dx + f(m_{23})(x_3 - m_{23}) , \text{ and } \]

\[
B = \int_{x_3}^{m_{34}} f(x) dx - f(m_{34})(m_{34} - x_3).
\]
Thus, $A$ is the difference between the area of the rectangle between $m_{23}$ and $x_3$ and the integral of the probability density from $m_{23}$ to $x_3$; $B$ is the difference between the integral of the probability density from $x_3$ to $m_{34}$ and the area of the rectangle between $x_3$ and $m_{34}$ (see Figure 3).

By a similar argument (see Figure 3),
\[
\frac{\partial U_2}{\partial x_2} = A' - B', \tag{5b}
\]
where
\[
A' = -\int_{x_2}^{m_{23}} f(x)dx + f(m_{23})(m_{23} - x_2), \quad \text{and}
\]
\[
B' = \int_{x_2}^{\infty} f(x)dx.
\]
Note that $\frac{\partial U_3}{\partial x_3}$ is independent of the location of $q_3$, as long as $q_3 \geq \bar{x}$; a similar statement holds for $\frac{\partial U_2}{\partial x_2}$.

PROPOSITION 2 (Effects of the Extreme Party in a 3-Party Scenario):
Assume the voter distribution is symmetric and unimodal, $f(0) > 0$, and the two mainstream parties are policy seeking based on the parliamentary mean, and that $q_2 \leq \bar{x}$ and $q_3 \geq \bar{x}$. If a right wing extreme party is added to this two-party scenario, both the mainstream parties move to the left. Similarly, if the extreme party is left wing, the mainstream parties move to the right.

PROOF: See the Appendix.

Intuitively, in the presence of the right-wing extreme party, any rightward movement by the center-right party draws vote share from the extreme right party, thus pushing the parliamentary mean leftward (because the center-right party exerts a weaker pull per vote on the parliamentary mean than does the extreme-right party). At the same time such rightward movement by the center-right party would add vote share to the center-left party, so both of these effects by themselves decrease the center right party's
utility. But by moving right, the center-right party (Party 3) also gains utility because $x_3$ and hence $p_3 x_3$ are increasing, which by itself would shift the parliamentary mean to the right, increasing the center-right party's utility. These countervailing effects create a trade-off and lead to an equilibrium location between the center point and Party 3's ideal point.

Why is the 3-party equilibrium position for Party 3 to the left of the two-party equilibrium position? To see this, as noted above, when there are three parties, the shift of votes from the extreme-right party to the center-right party when the latter moves right pulls the parliamentary mean to the left, but with only two parties, this effect, of course, does not occur. The other two effects (losing votes to the center-left party and increase of $p_3 x_3$) are the same in either a 2-party or 3-party scenario. Thus, rightward movement by the center-right party relative to its 2-party equilibrium decreases its utility, while, conversely, moving left increases that utility. Hence, the equilibrium for the center-right party in a 3-party scenario is to the left of its equilibrium in a 2-party scenario.

20 These two effects decrease Party 3’s utility in proportion to the sum of

$$f(m_{34})*(m_{34} - x_3) \text{ and } f(m_{23})*(x_3 - m_{23}).$$

21 This increase in Party 3’s utility is proportional to the integral of $f$ between $m_{23}$ and $m_{34}$.

22 The net change in party 3’s utility when that party moves left is proportional to $A - B$ in Figure 3 (see also the proof of Proposition 2).

23 In the dominant-party model, the net effect of rightward movement by the center-right party is similar, except that drawing votes away from the extreme-right party benefits Party 3 if its ideal position is closer to $x_3$ than to $x_4$ and in any event does not disadvantage Party 3 as much as in the parliamentary-mean model [the term $f(m_{34})*(m_{34} - x_3)$ is replaced by $f(m_{34})*(q_3 - m_{34})$]. Accordingly, in the dominant-party model, Party 3 is motivated to move left from its 2-party equilibrium location, but by less than in the parliamentary-mean model.
Now we turn to the center-left party (Party 2). Why is the 3-party equilibrium for this party also further to the left than in the 2-party model? To see this, note that when the center-right party moves left (in reaction to the extreme-right party), the cut-point \( m_{23} \) between the two mainstream parties also moves left away from the point where the density peaks, so the center-left party can better afford to lose support to the center-right party than in the 2-party scenario (i.e., it loses less votes by moving left). Since its benefit by gaining leftward position (the \( p_2 x_2 \) term) is the same as in the 2-party model, movement left is a net benefit, until a 3-party equilibrium is reached.

**PROPOSITION 3 (Effects of Extreme Parties in a 4-Party Scenario):**
Assume the voter distribution is symmetric and unimodal, \( f(0) > 0 \), and the two mainstream parties are policy seeking based on the parliamentary mean, and that \( q_2 \leq \bar{x} \) and \( q_3 \geq \bar{x} \). If we add to this two-party situation two extreme parties at fixed positions, one to the left and one to the right, then both mainstream parties move inward, i.e., become less extreme.

**PROOF:** Casamatta and De Donder (2005) show that for a symmetric voter density, log concavity of \( F(x) \) is sufficient for a four-party equilibrium (i.e., in which all four parties including the extreme parties are free to move) and provide a recursive formula for the party positions at equilibrium. The form of this equilibrium implies that the mainstream parties moderate their positions relative to their two-party equilibrium positions. Our proof of Proposition 3, for the slightly more general class of symmetric unimodal voter distributions, under the assumption that the positions of the extreme parties are fixed, is similar to that for Proposition 2 and is given in the Appendix.

Next, in a context not considered by Casamatta and De Donder (2005), we show that if the voter distribution is sufficiently bimodal, i.e., polarized instead of unimodal, then the expectations are quite different: the mainstream parties may be motivated to diverge to extreme positions.
PROPOSITION 4: Divergence of Equilibria for Bimodal Voter Distributions.

Assume that the voter density \( f \) is symmetric and bimodal, \( f(0) > 0 \), and that \( c \) denotes the least value greater than the right-hand mode for which \( f(c) = f(0) \) (see the example in Figure 4). In the four-party model, if \( x_3 \) and \( x_2(= -x_3) \) constitute a Nash equilibrium, and \( x_1 < x_2, x_4 > x_3 \), then

\[
x_3 \geq c + \frac{h}{f(0)},
\]

where \( h = \int_0^c [f(x) - f(0)] \, dx \) is the area of (either) hump.

PROOF: See the Appendix.

Intuitively, the proposition says that in order for a Nash equilibrium to occur, the mainstream parties would need to be separated by a factor that increases with the size of the humps (that define each mode) and inversely related to the density at the center point of the scale. Thus, for example, if the voter distribution is the mixture of two equally weighted normal distributions with the same variance but means separated by three standard deviations,\(^{24}\) and if \( x_3 \) and \( x_2(= -x_3) \) constitute a Nash equilibrium, then, by Proposition 5, \( x_3 / \sigma \geq 2.43 + 0.178/0.130 = 3.80 \), so \( x_3 \) would need to be nearly four standard deviations from the mean (see Figure 4). Accordingly, the mainstream parties would have to be exceedingly extreme to achieve equilibrium, so equilibrium under this circumstance would be correspondingly unlikely. In general, for bimodal distributions, the mainstream equilibrium locations rapidly diverge from one another as bimodality becomes more pronounced.

Finally, we investigate the existence and nature of Nash equilibria in the three- and four-party models for fixed extreme parties.

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\(^{24}\) For such a mixed normal to be bimodal, the means of the two constituent normal distributions, each with standard deviation \( \sigma \), must be separated by more than \( 2\sigma \) (Behboodian, 1970).
**PROPOSITION 5: Policy-seeking Nash Equilibrium.** Assume that the voter density, \( f \), is symmetric and unimodal, \( f(0) > 0 \), and that \( \partial U_3 / \partial x_3 \) is continuous as a function of \( x_2 \) and \( x_3 \). Furthermore, if recursive application of equations (4) leads to sequences of values of \( x_3 \) that converge to a limit \( x_3^* \) (and \( x_2^* \) is defined as \( -x_3^* \)), then \( (x_2^*, x_3^*) \) constitute a policy-seeking Nash equilibrium.\(^{25}\)

**PROOF:** See the Appendix.

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**4. Model Robustness: Effects of Alternative Assumptions on the Locations of Nash Equilibria**

In this section, we note the effects of several alternative assumptions; details are in the Appendix. First, suppose that mainstream parties consider the extreme parties as pariahs that could not be considered for a coalition government under any circumstances, so that the two mainstream parties evaluate their utility of the election outcome entirely in terms of their own locations. Tests of this scenario for normal voter distributions indicate that the mainstream parties neither moderate nor do they become more extreme at equilibrium. Intuitively, discounting the contribution of an extreme right party to policy formation annuls any value that the extreme party might provide to compensate for the mainstream party's loss of vote share to that extreme party.

Second, under an assumption that citizens located at relatively extreme locations abstain if no party is located near their position, we suggest by example that for plausible levels of abstention, it appears likely that mainstream parties would be motivated to moderate their strategies when extreme parties materialize, just as when there was no abstention.

\(^{25}\) For a uniform voter distribution, \( \partial U_3 / \partial x_3 \) is constant, so all locations for \( x_2 \) and \( x_3 \) that are more moderate than the mainstream parties' ideal points are trivially Nash equilibria.
Third, we find little effect on equilibrium positions when the extreme parties, as well as the mainstream parties, can change location.

Fourth, adding an independent office-seeking component exerts a strong moderating effect on optimal strategies of mainstream parties, drawing them toward the center point of the scale, whether or not extreme parties impinge. However, mainstream parties are drawn to complete convergence with a smaller admixture of vote share when extreme parties are present than when they are not. Finally, we explore in the Appendix the effects on equilibria of alternative values of the fixed positions of the extreme parties or if seat share is super-proportional relative to vote share.

III. Discussion

We have determined Nash equilibrium strategies for a pair of policy-seeking, mainstream parties under a PR system, on the flank of one of which, an extreme party enters. Under the assumption that the voter density is symmetric and unimodal and the policy outcome is interpreted as the parliamentary mean, we find that at equilibrium both mainstream parties move away from the new extreme party. If two extreme parties enter on each flank, we extend -- to any symmetric, unimodal voter density -- the result of Cassamata and De Donder (2005) that both mainstream parties moderate at equilibrium. Furthermore, we show that, under certain conditions, our results continue to hold when the policy outcome is determined by one dominant party.²⁶

On the other hand, we prove for the scenario with extreme parties on both ends of the spectrum that, if the voter density is significantly bimodal, any equilibria for the mainstream parties are rapidly divergent as polarization increases. Finally, examples suggest that if the mainstream parties discount extreme party potential to be part of government, their equilibrium positions are little affected by the entry of those extreme parties.

²⁶ If an issue salient to an extreme party is considered as a separate dimension, a mainstream party may change on the new dimension rather than on the left right dimension (Abou-Chadi and Krause, personal communication 2018). Pursuing this avenue is left to further research.
Alonso and Fonseca (2012), in their comparison of party systems with and without the presence of extreme right parties that take a strong anti-immigration position, find that Christian Democratic parties were less anti-immigration when extreme right parties were present than when they were not. This finding accords with our Proposition 2, which indicates that, under our assumptions, the introduction of an extreme party on the right should motivate the proximate mainstream right party to moderate its stand.

Mainstream party movement in the direction opposite to that of a burgeoning extreme party is suggested by three recent presidential elections: that of Alexander Van der Bellen as President of Austria over far-right Freedom Party candidate Norbert Hofer in December, 2016, and in the elections of Jacques Chirac in 2002 and Emmanuel Macron in 2017 as President of France.

Florian Bieber of the University of Graz, Austria, in a *New York Times* op-ed (Bieber, 2016) suggests that the following lesson can be drawn from the 2016 Austrian presidential election:

Catering to the Freedom Party is likely to be a losing strategy for the centrists. One lesson from the presidential election is that presenting a real alternative is the way to defeat the far right. ... Instead of pandering to the far right, Mr. Van der Bellen presented himself as a clear alternative.\(^{27}\)

In the French presidential elections of 2002, the success of Jean-Marie Le Pen in entering the second round of the French presidential election led to a consensus repudiation of his politics by the left (denied a place on the second round because of fragmentation of the left-leaning vote), as well as by the right -- in the person of Jacques Chirac. While in 2017, Emmanuel Macron, on the center right, successfully took a stance clearly in opposition to the Le Peniste attacks on the European Union and its immigration policies.

\(^{27}\) Van der Bellen's status as a Green may have attracted some support from voters disenchanted with the establishment parties, which augmented his appeal.
References


Table 1. Policy-seeking Nash equilibrium for mainstream parties, with two, three, and four party contests, assuming a normal electorate.

a. Parliamentary mean model where the ideal points of the mainstream parties straddle the parliamentary mean ($q_2 \leq \bar{x} \leq q_3$)

<table>
<thead>
<tr>
<th></th>
<th>Extreme left party</th>
<th>Mainstream left party</th>
<th>Mainstream right party</th>
<th>Extreme right party</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$x_3$</td>
<td>$x_4$</td>
</tr>
<tr>
<td>2 parties</td>
<td>-0.627 (50.0%)</td>
<td>0.627 (50.0%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 parties</td>
<td>-0.777 (38.2%)</td>
<td>0.477 (54.8%)</td>
<td>1 (7.0%)</td>
<td></td>
</tr>
<tr>
<td>4 parties</td>
<td>-1 (7.7%)</td>
<td>-0.427 (42.3%)</td>
<td>0.427 (42.3%)</td>
<td>1 (7.7%)</td>
</tr>
</tbody>
</table>

Note: Voters are normally distributed, with mean 0 and standard deviation 0.5. Extreme candidates, if present, are fixed at $x_1 = -1$ and $x_4 = 1$. These equilibria apply not only for the parliamentary mean model when $q_2 \leq \bar{x} \leq q_3$, but also to the dominant party model if $q_2 \leq x_1$ and $q_3 \geq x_4$. For the 4-party scenario, it is assumed that at equilibrium, $q_2 = -q_3$ (this is the case for the 2-party scenario by Proposition 1). Values in parentheses are the vote shares at equilibrium.
Table 1 (continued)

b. Dominant party model when mainstream party ideal points are less extreme than the extreme parties, but more extreme than the 2-party equilibrium locations

<table>
<thead>
<tr>
<th></th>
<th>Extreme left party $x_1$</th>
<th>Mainstream left party $x_2$</th>
<th>Mainstream right party $x_3$</th>
<th>Extreme right party $x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 parties</td>
<td>-0.627 (50.0%)</td>
<td>0.627 (50.0%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 parties</td>
<td>-0.704 (43.9%)</td>
<td>0.550 (50.1%)</td>
<td>1 (6.1%)</td>
<td></td>
</tr>
<tr>
<td>4 parties</td>
<td>-1 (6.2%)</td>
<td>-0.540 (43.8%)</td>
<td>0.540 (43.8%)</td>
<td>1 (6.2%)</td>
</tr>
</tbody>
</table>

Note: Mainstream party ideal points are located at ±0.8. Voters are normally distributed, with mean 0 and standard deviation 0.5. For the 4-party scenario, it is assumed that at equilibrium, $q_2 = -q_3$. Values in parentheses are the vote shares at equilibrium.
Figure 1. Effect of entrance of extreme parties on Nash equilibria of mainstream parties

Note: The arrows indicate the movement of mainstream parties from their equilibrium locations in a 2-party scenario to their equilibrium locations in either a 3-party or a 4-party scenario. Note that in each case, each mainstream party moves away from the extreme party that has entered nearest its position. The voter distribution is assumed normal, with mean 0 and standard deviation 0.5. The 2-party scenario (no extreme parties) features left and right mainstream parties. The 3-party scenario adds a single extreme party on the right, fixed at location 1.0. The 4-party scenario adds extreme parties on the left and on the right, fixed at -1.0 and 1.0, respectively. The equilibria apply either for the parliamentary mean model when \( q_2 \leq \bar{x} \leq q_3 \), or to the dominant party model if \( q_2 \leq x_i \) and \( q_3 \geq x_4 \). A symmetric voter distribution, which is also unimodal for the 3- and 4-party scenarios, is assumed.
Figure 2. Utility maximization under parliamentary-mean and dominant-party models.

b. Utility for Center Right Party in Three-Party Scenario

Note: For a three-party scenario, utility for the center-right party is plotted against the party's announced position, for both the parliamentary-mean model (solid line) and dominant-party model (dashed line). Voters are assumed normally distributed with mean 0 and standard deviation 0.5. Announced positions are fixed at $x_2 = -0.5$, and $x_4 = +1$; the desired location of the center-right party (Party 3) is set at 0.8. Utility under the dominant party model drops rapidly when the center-right party's announced position exceeds its own desired location.

The arrows indicate the equilibrium values for the center-right party, under the parliamentary-mean (solid arrow) and dominant-party (dashed arrow) models, respectively. (Note that the parliamentary-mean curve in this example does not reach its maximum at the equilibrium value for the center-right party because the center-left party is not at its equilibrium value of -0.777.) For comparison, the vertical line indicates the center-right party's equilibrium position when there are only two parties.
Figure 3. Three-party scenario depicting $\partial U_3/\partial x_3$ and $\partial U_2/\partial x_2$

Note that $\partial U_3/\partial x_3$ is the area under the curve (between $m_{23}$ and $m_{34}$) minus the area of the rectangles between $m_{23}$ and $x_3$ and between $x_3$ and $m_{34}$. Because $m_{23} \geq 0$, it follows that $\partial U_3/\partial x_3 = B - A$. $\partial U_2/\partial x_2$ is the area of the rectangle between $x_2$ and $m_{23}$ minus the area under the curve from $-\infty$ to $m_{23}$. It follows that $\partial U_2/\partial x_2 = A' - B'$ where, because $m_{23} > 0$ in this example, $A'$ is the difference in area of two regions. A parliamentary mean model with $q_2 \leq \bar{x} \leq q_3$ or dominant party model with $q_3 \geq x_4$ is assumed.
Figure 4. Mixed normal voter distribution with extreme Nash equilibria

Note: The mixed-normal voter distribution is specified by \( f(x) = 0.5 * f_1(x) + 0.5 * f_2(x) \), where \( f_1 \) is normal with mean \( -\mu \) and standard deviation \( \sigma \) and \( f_2 \) is normal with mean \( +\mu \) and standard deviation \( \sigma \). In this example \( \sigma = 1 \) and \( \mu = 1.5 \). If in the four-party scenario, \( x_2 \cong -3.80 \) and \( x_3 \cong 3.80 \) (and provided that \( x_1 \) and \( x_4 \) are more extreme than these mainstream parties), then a Nash equilibrium occurs, but not for more moderate locations for the mainstream parties.
Appendix to
“What are the Effects of Entry of New Extremist Parties on the Policy Platforms of Mainstream Parties?”

PROPOSITION 1: In a model consisting of only the two mainstream parties, assume that they are policy seeking based on the parliamentary mean, \( q_2 \leq \bar{x} \) and \( q_3 \geq \bar{x} \). Assume further that the voter density, \( f \), is symmetric and \( f(0) > 0 \). Then a unique Nash equilibrium occurs for \( x_2 = -1/(2f(0)) \) and \( x_3 = +1/(2f(0)) \).\(^{28}\)

PROOF: First we show that if a Nash equilibrium exists for which \( x_2 \) and \( x_3 \) are symmetric about 0, then it is of the form specified. Note that

\[
U_3 = p_2x_2 + p_3x_3 - q_3 = F(m_{23})x_2 + \left[1 - F(m_{23})\right]x_3 - q_3, \text{ so that}
\]

\[
\frac{\partial U_3}{\partial x_3} = x_2f(m_{23})/2 + \left[1 - F(m_{23})\right] - x_3f(m_{23})/2
\]

\[
= -f(m_{23})(x_3 - x_2)/2 - F(m_{23}) + 1 = 0,
\]

because \( x_2 \) and \( x_3 \) constitute a Nash equilibrium. By the symmetry assumption above, \( m_{23} = 0 \), so that \( F(m_{23}) = 1/2 \). We conclude that

\[
f(0)(x_3 - x_2)/2 = 1/2, \text{ so that}
\]

\[
x_3 = 1/(2f(0)), \text{ and similarly, } x_2 = -1/(2f(0)). \tag{A2}
\]

\(^{28}\)Brams and Merrill (1991) obtain (and Adams and Merrill, 2006, cite) a similar-looking formula for a two-party equilibrium in which parties seek to maximize the probability that they obtain the vote of the median voter. In their case, however, the density function \( f \) represents the parties' uncertainty about the location of the median voter, whereas in the present case \( f \) represents the distribution of voters.
Conversely, we show that $x_2 = -1/(2f(0))$ and $x_3 = +1/(2f(0))$ constitute a Nash equilibrium. If $x_2$ is fixed at $-1/(2f(0))$, then

\[
\frac{\partial U_2}{\partial x_3} \bigg|_{x_3 = 1/(2f(0))} = -f(0)(1/f(0))/2 - F(0) + 1 = 0 \text{ by equation (A1) above. Furthermore,}
\]

\[
\frac{\partial^2 U_2}{\partial x_2^2} \bigg|_{x_3 = 1/(2f(0))} = -f(m_{23}) - (1/4)f'(m_{23})(x_3 - x_2). \text{ Thus,}
\]

\[
\frac{\partial^2 U_3}{\partial x_2^2} \bigg|_{x_3 = 1/(2f(0))} = -f(0) - (1/4)f''(0)(1/f(0)) = -f(0) < 0 \text{ because } f''(0) = 0 \text{ (by symmetry of the voter density) and } f(0) > 0. \text{ It follows that } U_3 \text{ has a maximum at } 1/(2f(0)) \text{ (and a similar result holds for } U_2 \text{ with } x_3 \text{ fixed), so that these formulas in equation (A2) define a Nash equilibrium.}
\]

Next we show that if $x_2$ and $x_3$ are not symmetrically located, i.e., that $m_{23} \neq 0$, then they do not constitute a Nash equilibrium. Suppose by way of contradiction, that $\bar{x}_2$ and $\bar{x}_3$ do constitute a Nash equilibrium for which $m_{23} = \bar{x}_2 + \bar{x}_3 / 2 \neq 0$. Without loss of generality, assume that $m_{23} < 0$. Then $\bar{x}_3$ is a solution of the equation

\[
\frac{\partial U_1}{\partial x_3} = -f(m_{23})(x_3 - x_2)/2 - F(m_{23}) + 1 = 0, \text{ so that}
\]

\[
\bar{x}_3 = m_{23} + \frac{1 - F(m_{23})}{f(m_{23})} \quad \text{and} \quad \bar{x}_2 = m_{23} - \frac{1 - F(m_{23})}{f(m_{23})}.
\]

Since $\frac{\partial U_2}{\partial x_2} = f(m_{23})(x_3 - m_{23}) - F(m_{23})$, we have

\[
\frac{\partial U_2}{\partial x_2} \bigg|_{x_3 = \bar{x}_2} = f(m_{23})(\bar{x}_3 - m_{23}) - F(m_{23}) = f(m_{23}) \left[ \frac{1 - F(m_{23})}{f(m_{23})} \right] - F(m_{23}).
\]

\[
= 1 - 2F(m_{23}) > 0
\]

Hence $U_2$ is strictly increasing at $\bar{x}_2$, so that $\bar{x}_2$ and $\bar{x}_3$ do not constitute a Nash equilibrium. We conclude that the Nash equilibrium specified in the Proposition is unique. q.e.d.
**Lemma 1:** Assume that the two mainstream parties are policy seeking based on the parliamentary mean, $q_2 \leq \bar{x}$ and $q_3 \geq \bar{x}$. If there is one extreme party $x_4$ on the right (the three-party model), then the rate of change of utility of each mainstream party with respect to that party's location is given by

$$\frac{\partial U_3}{\partial x_3} = \int_{m_{23}}^{m_{34}} f(x)dx - [f(m_{23})(x_3 - m_{23}) + f(m_{34})(m_{34} - x_3)]$$

and

$$\frac{\partial U_2}{\partial x_2} = -\int_{0}^{m_{23}} f(x)dx + f(m_{23})(m_{23} - x_2).$$

(A3a)

If there are two extreme parties $x_1$ and $x_4$, the first on the left and the latter on the right (the four-party model), then the rate of change of utility of each mainstream party with respect to that party's location is given by

$$\frac{\partial U_1}{\partial x_3} = \int_{m_{21}}^{m_{34}} f(x)dx - [f(m_{21})(x_3 - m_{21}) + f(m_{34})(m_{34} - x_3)]$$

and

$$\frac{\partial U_2}{\partial x_2} = -\int_{m_{12}}^{m_{23}} f(x)dx + [f(m_{12})(x_2 - m_{12}) + f(m_{23})(m_{23} - x_2)].$$

(A3b)

Assume further that the voter density, $f$, is symmetric and unimodal. If $\partial U_3/\partial x_3$ is continuous, then, for any fixed value of $x_2$, $U_3$, reaches a maximum at some point $x_3$ on the interval from 0 to $x_4$ and this point satisfies the recursive equation

$$x_3 = \frac{p_3 + m_{23}f(m_{23}) - m_{34}f(m_{34})}{f(m_{23}) - f(m_{34})}.$$  

(A4)

Similar recursive equations for maximum points hold for $x_2$ and $x_3$ in the 3-party and 4-party scenarios.

**Proof:** We derive the first formula in (A3a). The other derivations for (A3a) and (A3b) are similar. Utility for party 3 is:
\[
U_3 = -|q_3 - \bar{X}| = -|q_3 - \frac{4}{3} \sum p_i x_i | \\
= F(m_{23}) x_2 + F(m_{34}) x_3 - F(m_{23}) x_3 + x_4 - F(m_{34}) x_4 - q_3 \\
= x_4 - F(m_{23})(x_3 - x_2) - F(m_{34})(x_4 - x_3) - q_3
\]

where \( m_{ik} \) is the midpoint between \( x_i \) and \( x_k \). It follows that

\[
\frac{\partial U_3}{\partial x_3} = -F(m_{23}) - \frac{1}{2} f(m_{23})(x_3 - x_2) + F(m_{34}) - \frac{1}{2} f(m_{34})(x_4 - x_3) \\
= p_3 - \frac{1}{2} [f(m_{23})(x_3 - x_2) + f(m_{34})(x_4 - x_3)] \\
= \int_{m_{23}}^{m_{34}} f(x)dx - \int [f(m_{23})(x_3 - m_{23}) + f(m_{34})(m_{34} - x_3)].
\]

To prove the second part of the proposition, recall that the voter density is assumed symmetric and unimodal. Note that, for \( x_3 = 0 \), \( \partial U_3 / \partial x_3 \geq 0 \) by applying equation (3a), (because, by unimodality of \( f \) , \( f(x) \geq f(m_{23}) \) for \( m_{23} \leq x \leq x_3 \) and \( f(x) \geq f(m_{34}) \) for \( x_3 \leq x \leq m_{34} \); see Figure A1, part a), whereas for \( x_3 = x_4 \), \( \partial U_3 / \partial x_3 \leq 0 \), (again, by unimodality of \( f \); see Figure A1, part b). Thus since \( \partial U_3 / \partial x_3 \) is continuous, then \( \partial U_3 / \partial x_3 = 0 \) for some \( x_3' \) between 0 and \( x_4 \). Since \( \partial U_3 / \partial x_3 \big|_{x_3 = 0} \geq 0 \) and \( \partial U_3 / \partial x_3 \big|_{x_3 = x_4} \leq 0 \), \( U_3 \) must have a maximum at one such point \( x_3' \) between 0 and \( x_4 \) that is a solution of \( \partial U_3 / \partial x_3 = 0 \). Solving the first equation in (A3a) for \( x_3 \), we obtain the three-party recursive equation in (A4). Derivation of the four-party recursive equation is similar. q.e.d.

<<< Figure A1 about here >>>

**PROPOSITION 2 (Effects of Extreme Party in a 3-Party Scenario):** Assume the voter distribution is symmetric and unimodal, \( f(0) > 0 \), and the two mainstream parties are policy seeking based on the parliamentary mean, and that \( q_2 \leq \bar{X} \) and \( q_3 \geq \bar{X} \). If a right wing extreme party is added to this two party scenario, both the mainstream
parties move to the left. Similarly, if the extreme party is left wing, the mainstream parties move to the right.

**PROOF:** For a 2-party scenario, by Proposition 1, \( x_2 = -1/(2f(0)) \) and \( x_3 = +1/(2f(0)) \) constitute a unique Nash equilibrium. Denote these equilibrium locations by \( x_2(2) \) and \( x_3(2) \), so that, in particular, \( x_2(2) = -1/(2f(0)) \) and \( x_3(2) = +1/(2f(0)) \); in turn, \( m_{23} = 0 \) in this 2-party scenario. Suppose that \( x_3 \) is set at \( x_3(2) \) and a right wing extreme party is added to the scenario at \( x_4 \). As before, the voter distribution has cumulative distribution function \( F \) and density function \( f \). Then,

\[
\frac{\partial U_3}{\partial x_3} = -A + B, \quad \text{where} \quad A = -\int_{m_{23}}^{x_3} f(x)dx + f(m_{23})(x_3 - m_{23}),
\]

and

\[
B = \int_{x_3}^{m_{34}} f(x)dx - f(m_{34})(m_{34} - x_3), \quad \text{are both non-negative (because of unimodality of } f \),
\]

(see equations (5a) and Figure 3, both in the main text), so we have

\[
A = f(0) \star x_3(2) - \left[ F(x_3(2)) - 0.5 \right]
\]

\[
= f(0) \star \frac{1}{2f(0)} - F(x_3(2)) + 0.5
\]

\[
= 1 - F(x_3(2)) = \int_{x_3(2)}^{x} f(x) \, dx,
\]

and

\[
B = \int_{x_3(2)}^{m_{34}} f(x)dx - f(m_{34})(m_{34} - x_3) \leq \int_{x_3(2)}^{x} f(x) \, dx.
\]

Thus,

\[
A \geq B.
\]

Because \( \frac{\partial U_3}{\partial x_3} = B - A \), it follows that

\[
\frac{\partial U_3}{\partial x_3} \leq 0 \quad \text{at} \quad x_3 = x_3(2),
\]

so party 3 gains utility (or does not lose utility) by moving to the left of \( x_3(2) \).\(^{29}\)

\(^{29}\)This inequality is strict unless party 4 is so far to the right that the midpoint between \( x_3(2) \) and \( x_4 \) lies to the right of the entire voter distribution, a highly atypical situation.
By equations (5b) (see also Figure 3), both in the main text, \( \frac{\partial U_2}{\partial x_2} = A' - B' \). As long as \( x_2 \) and \( x_3 \) are in their 2-party-scenario equilibrium positions, \( m_{23} = 0 \) and \( \frac{\partial U_2}{\partial x_2} = A' - B' = 0 \). But if \( x_1 \) moves to the left, so does \( m_{23} \) and the quantity \( A' \) declines (because \( f \) is increasing on the left of 0), so that at \( x_2(2), \frac{\partial U_2}{\partial x_2} = A' - B' \leq 0 \). Thus party 2 gains utility (or does not lose utility) by moving to the left of \( x_2(2) \). q.e.d.

**PROPOSITION 3 (Effects of Extreme Parties in a 4-Party Scenario):**
Assume the voter distribution is symmetric and unimodal, \( f(0) > 0 \), and the two mainstream parties are policy seeking based on the parliamentary mean, and that \( q_2 \leq \bar{x} \) and \( q_3 \geq \bar{x} \). If we add to this two party situation two extreme parties at fixed positions, one to the left and one to the right, then both mainstream parties move inward, i.e., become less extreme.

**PROOF:** By Proposition 1, given two (mainstream) parties, equilibrium occurs for \( x_2 = -1/(2f(0)) \) and \( x_3 = +1/(2f(0)) \). Denote these values by \( x_2(2) \) and \( x_3(2) \), to indicate that they are equilibrium locations for a 2-party scenario, so that, in particular, \( x_2(2) = -1/(2f(0)) \) and \( x_3(2) = +1/(2f(0)) \). With the addition of extreme parties 1 and 4, located equidistant from 0 and on either side, we can assume that the new equilibrium positions of parties 2 and 3, to be denoted by \( x_2(4) \) and \( x_3(4) \), are also equidistant from 0 and on either side of 0, so that \( m_{23} = 0 \) (for either the 2-party or 4-party scenario). As before, the voter distribution has cumulative distribution function \( F \) and density function \( f \). If \( x_4 \) is set as \( x_3(2) \) then (see Figure 3 in the main text, noting that the following equation is not affected by the presence of party 1),

\[
A = f(0) * x_3(2) - [F(x_3(2)) - 0.5]
\]
\[ f(0)^* \frac{1}{2f(0)} - F(x_3(2)) + 0.5 \]
\[ = 1 - F(x_3(2)) = \int_{-\infty}^{x_3(2)} f(x) \, dx, \]
and
\[ B \leq \int_{x_3(2)}^{\infty} f(x) \, dx. \]

Thus,
\[ A \geq B. \]

Because \( \frac{\partial U_1}{\partial x_3} = B - A \), it follows that
\[ \frac{\partial U_3}{\partial x_3} \leq 0 \text{ at } x_3 = x_3(2), \]
so party 3 gains utility (or does not lose utility) by moving to the left of \( x_3(2) \). Because of the symmetry, a similar argument applies to party 2, so that \( \frac{\partial U_2}{\partial x_2} \geq 0 \text{ at } x_2 = x_2(2). \)

Thus, party 2 gains utility (or does not lose utility) by moving to the right of \( x_2(2) \).

Finally, note that inequality \( \frac{\partial U_3}{\partial x_3} \leq 0 \) is strict unless party 4 is so far to the right that the midpoint between \( x_3(2) \) and \( x_4 \) lies to the right of the entire voter distribution, a highly atypical situation. A similar statement holds concerning \( \frac{\partial U_2}{\partial x_2} \geq 0 \). q.e.d.

**PROPOSITION 4: Divergence of Equilibria for Bimodal Voter Distributions.**

Assume that the voter density \( f \) is symmetric and bimodal, and that \( c \) denotes the least value greater than the right-hand mode for which \( f(c) = 0 \) (see Figure A2). In the four-party model, if \( x_3 \) and \( x_2 (= -x_3) \) constitute a Nash equilibrium, and \( x_1 < x_2, x_4 > x_3 \), then
\[ x_3 \geq c + \frac{h}{f(0)}, \quad (A5) \]
where \( h = \int_0^x [f(x) - f(0)] \, dx \) is the area of (either) hump.

**PROOF:** Note first that \( U_3 \) may have a critical point between 0 and \( c \), but such a critical point defines a minimum, not a maximum for \( U_3 \). For \( U_3 \) to have a critical point maximum at a point \( x_3 \) greater than \( c \), requires that

\[
\frac{\partial U_3}{\partial x_3} = \int_0^{x_3} f(x) \, dx - f(0)[x_3 - 0] + \int_{x_3}^{m_{34}} f(x) \, dx - f(m_{34})[m_{34} - x_3] = 0, \tag{A6}
\]

since \( m_{33} = 0 \). We will construct a value \( \overline{x} \) that is greater than or equal to \( c + \frac{h}{f(0)} \) and such that \( x_3 \) must be greater than \( \overline{x} \) for equation (A6) to hold. That will show that \( x_3 \geq c + \frac{h}{f(0)} \). To do this, because \( f \) is decreasing to the right of \( c \), we can choose \( \overline{x} \) greater than \( c \) so that

\[
\int_0^{\overline{x}} f(x) \, dx = f(0)\overline{x}. \tag{A7}
\]

We first show that \( x_3 \geq \overline{x} \). Again, because \( f \) is decreasing to the right of \( c \), the term \( \int_{x_3}^{m_{34}} f(x) \, dx - f(m_{34})[m_{34} - x_3] \geq 0 \), which implies that for \( \frac{\partial U_3}{\partial x_3} = 0 \) (that is, for equation (A6) to hold), the term \( \int_0^{x_3} f(x) \, dx - f(0)x_3 \) must be non-positive. But the function \( g \), defined by \( g(x) = \int_0^x f(t) \, dt - f(0)x \), is decreasing in \( x \) and \( g(x_3) \leq 0 = g(\overline{x}) \) by equation (A7), so that \( x_3 \geq \overline{x} \). Finally, equation (A7) can be rephrased as

\[
\int_0^\overline{x} [f(x) - f(0)] \, dx = \int_0^\overline{x} [f(0) - f(x)] \, dx, \quad \text{or equivalently, using the definition of } h, \text{ as}
\]

\[
h = \int_0^\overline{x} [f(0) - f(x)] \, dx, \quad \text{which in turn becomes } f(0)(\overline{x} - c) = h + \int_0^{\overline{x}} f(x) \, dx, \quad \text{so that}
\]

dropping the non-negative term \( \int_0^\overline{x} f(x) \, dx \), we obtain

\[
\overline{x} \geq c + \frac{h}{f(0)}, \quad \text{so } x_3 \geq c + \frac{h}{f(0)}. \quad \text{q.e.d.}
\]

<<<< Figure A2 about here >>>>
PROPOSITION 5 (Policy-seeking Nash Equilibrium): Assume a four-party model for which the voter density, \( f \), is symmetric and unimodal and that \( \partial U_3 / \partial x_3 \) is continuous as a function of \( x_2 \) and \( x_3 \). Furthermore, if recursive application of equation (A4) leads to sequences of values of \( x_3^* \) that converge to a limit \( x_3^* \) (and \( x_2^* \) is defined as \(- x_3^*\)), then \((x_2^*, x_3^*)\) constitute a policy-seeking Nash equilibrium.\(^{30}\)

PROOF: To be specific, define \( x_2^{(0)} = x_3^{(0)} = 0 \), define \( x_3^{(i)} \) by equation (A4) and specify \( x_2^{(i)} = -x_3^{(i)} \). Recursively, define \( x_3^{(n)} \) by equation (A4) and \( x_2^{(n)} = -x_3^{(n)} \). The hypothesis states that \( x_3^{(n)} \) converges to a limit, \( x_3^* \), and \( x_2^* = -x_3^* \). By equation (A4), \( \partial U_3 / \partial x_3 = 0 \) when \( x_2 = x_2^{(n-1)} \) and \( x_3 = x_3^{(n)} \), for all positive \( n \). Thus, by Lemma 1, \( U_3 \) has a maximum at \( x_3 = x_3^{(n)} \) for given \( x_2 = x_2^{(n-1)} \). By the continuity of \( \partial U_3 / \partial x_3 \),

\[
\frac{\partial U_3}{\partial x_3} \bigg|_{x_2 = x_2^*, x_3 = x_3^*} = \lim_{n \to \infty} \frac{\partial U_3}{\partial x_3} \bigg|_{x_2 = x_2^{(n-1)}, x_3 = x_3^{(n)}} = 0.
\]

Thus, \( U_3 \) has a maximum at \( x_3 = x_3^* \), given \( x_2 = x_2^* \) (and for \( x_2 = x_2^* \), given \( x_3 = x_3^* \)). q.e.d.

**Contrast between the Calvert (1985) model and the parliamentary-mean model.**

Calvert (1985) obtains convergence to the median under assumptions that are quite different from the parliamentary-mean model. He defines the goal of each party as maximizing its utility for the winning platform, for example, for Party 3, that is minimizing the quantity \( |q_3 - y| \), where \( y \) is the location of the (single) party who wins a

---

\(^{30}\) For a uniform voter distribution, \( \partial U_3 / \partial x_3 \) is constant, so all locations for \( x_2 \) and \( x_3 \) that are more moderate than the mainstream parties' ideal points are trivially Nash equilibria.
majority. Under the parliamentary-mean model, by contrast, Party 3 seeks to minimize $|q_3 - \bar{x}|$, where the parliamentary mean $\bar{x}$ weights the vote share (and hence the seat share) of both parties.

In the Calvert model, if, say, the center-left party is to the left of the median, the center-right party can move closer to the median and win with a platform to the right of the median. In the same situation under the parliamentary-mean model, movement by the center-right party closer to the median would move the parliamentary mean into negative territory as long as the two parties are not too far from the median.
Detailed Effects of Alternative Assumptions on the Locations of Nash Equilibria

1. What if the mainstream parties evaluate utility from the locations of only the mainstream parties?

As noted earlier, it may make sense, at least for some countries to explore an alternative utility function in which the mainstream parties evaluate their utility of the election outcome entirely in terms of the locations of the mainstream parties. This might make sense if the mainstream parties consider the extreme parties as pariahs that could not be considered for a coalition government under any circumstances. Accordingly, utility for party $j$ in the four-party model is given by

$$U_j'' = -q_j - \frac{(p_2 x_2 + p_3 x_3)}{(p_2 + p_3)}.$$

Under this alternative utility function, given a normal voter distribution with mean 0 and standard deviation 0.5, we investigated locations of the extreme right party anywhere between 0.75 and 1.5 (and the extreme left party between $-0.75$ and $-1.5$). Throughout this range, numerically evaluated Nash equilibria for the mainstream parties range from $-0.639$ to $-0.648$ for $x_2$ and $0.639$ to $0.648$ for $x_3$. These locations for the mainstream parties are comparable to their equilibrium strategies when no extreme parties are present and indicate none of the moderation that is projected when extreme parties do exist and are accounted for in mainstream party utilities. But under the present assumption, neither is the mainstream response to extreme party entry more extreme. Intuitively, when, say, a center-right party that discounts the contribution of an extreme right party to policy formation moderates, no value is provided to compensate for the mainstream party's loss of vote share to that extreme party. This reduces the motivation to moderate relative to a center-right party that does assign utility to the position of the party on its flank. If the mainstream parties only partly take into account the locations of
extreme parties in assessing their utility, then equilibrium positions are intermediate between those for $U$ and $U''$.31

2. What role might abstention play?

So far we have assumed that citizens' decisions whether to cast a vote are not affected by the locations of available parties. Suppose instead that citizens located at relatively extreme locations abstain if no party is located near their position. What qualitative effects would this assumption have on our results? Consider a scenario in which only the two mainstream parties are in the competition and they face the threat of abstention by extreme members of the potential electorate. To be specific, suppose that all citizens abstain if located at least a distance $d$ more extreme than the mainstream parties. We can show that at equilibrium the location of, say, the mainstream party on the right satisfies the recursive equation $x_3 = (2F(x_3 + d) - 1)/[2*f(0)]$.32 Thus, for a normal distribution with mean 0 and standard deviation 0.5, if there were no abstention, $x_3 = 0.627$, but with abstention, $x_3 = 0.626, 0.610, or 0.561$, if $d = 1, 0.5, or 0.25$, respectively. This example suggests that plausible levels of abstention do not greatly affect the mainstream strategies when no extreme parties are present. These strategies are still more extreme than the equilibrium strategies ($\pm 0.427$; see Table 1) that would occur if extreme parties do enter and attract the votes of citizens who would otherwise have abstained.

Thus, even with the threat of abstention when extreme parties are not available, it appears likely that mainstream parties would be motivated to moderate their strategies.

31 If a lottery over competing parties, rather than the parliamentary mean, is used to specify utility, then the effect of using only the mainstream parties to define utility is the same as when the parliamentary mean specification is employed, as long as the ideal points of the mainstream parties are at least as extreme as their declared positions.

32 Modifying slightly the proof of Lemma 1, we conclude that $x_3f(0) = \int_{x_3-d}^{x_3+d} f(x) \, dx$, so that $x_3 = (2F(x_3 + d) - 1)/[2*f(0)]$. 
when extreme parties do materialize, just as when there was no abstention. Accordingly, accounting for abstention does not change our results qualitatively.

3. What if the extreme parties are themselves mobile?

Next, we return to the utility function $U$ based on the parliamentary mean and on all parties, to investigate what happens when the extreme parties can also change location (while as before, we allow the mainstream parties to change location). Even when constraints on movement by the extreme parties are relaxed, the differences between this more general scenario and what we obtained when we held the positions of extremist parties fixed tend to be minor -- as shown in Table A.1. Intuitively, given policy-seeking incentives, the additional vote share gained from moderation by mobile extreme parties generally forfeits too much utility in moving away from the party’s most preferred policy position to make substantial movement in a centrist direction a viable option for extremist parties.33

<<< Table A.1 about here >>>

4. What if each mainstream party values its vote share per se in addition to its contribution to the parliamentary mean?

Although a party's vote share is a significant constituent of its policy-seeking utility, we can imagine that a party may value its vote share per se in addition to its effect on its policy-seeking utility. Valuation of vote share per se could be interpreted as an office-seeking motivation. Although it is difficult to determine a common scale on which

33 However, when a previously extreme party commits to a clearly centrist move, i.e., actually moderates its ideal point, it may be because its new leadership has made a conscious decision to reject a veto by its more extreme elements in the confidence that the new electorate gained by a more centrist move will provide a new and offsetting cadre of supporters. This may be the mechanism underlying Marine Le Pen’s decision to “evict” her father from the party that he founded (BBC, 2015)!
to precisely compare these two effects, adding an independent vote-share component
exerts a strong moderating effect on optimal strategies of mainstream parties, drawing
them toward the center point of the scale, whether or not extreme parties impinge.
However, mainstream parties are drawn to complete convergence with a smaller
admixture of vote share when extreme parties are present than when they are not.

5. What is the effect of alternative locations for the fixed positions of the extreme
party(s)? Calculation of Nash equilibria for the 3-party and 4-party models with normal
voter distributions.

For a parliamentary mean model with \( q_2 \leq \bar{x} \) and \( q_3 \geq \bar{x} \), for the two-party,
three-party, and four-party models and normal voter distributions (with standard
deviation = 0.5), Table A.2 depicts Nash equilibria, for a range of possible fixed positions
of the extreme parties, for both the parliamentary-mean and dominant-party models. As
the fixed locations for the extreme parties become more extreme (ranging from locations
one standard deviation from the mean to three standard deviations from the mean), the
Nash equilibria for the mainstream parties are correspondingly more extreme, and
concomitantly, their vote shares are substantially larger. Note that the patterns of
movement by mainstream parties upon entry by extreme parties are qualitatively similar
either when the ideal points of the mainstream parties are as extreme as the locations of
the extreme parties or when these ideal points are less extreme. The degree of movement
of mainstream positions in response to extreme-party entrance, however, is dampened in
the latter case.\(^{34}\) Finally, Table A.2 reports Nash equilibrium positions for the dominant
party model with \( q_2 > x_1 \) and \( q_3 < x_4 \).

\(^{34}\) Note, as before, that under the assumptions, the equilibrium location for \( x_3 \) is the same
for any \( q_3 \geq \bar{x} \), because \( \frac{\partial U_3}{\partial x_3} \) is independent of the location of \( q_3 \) as long as \( q_3 \geq \bar{x} \).
6. What if, in the dominant-party model, seat share is super-proportional, relative to vote share.

Suppose, in the dominant-party model, party influence on governmental policy is proportional not to vote share, but to seat share, i.e., relative to smaller parties, assume that larger parties have more influence on policy relative to smaller parties than their vote proportions would indicate.\textsuperscript{35} A simple model for this effect is to assume that seat share is proportional not to \( p_i \), but rather to \( p_i^2 \).\textsuperscript{36} Under this assumption, for a normally distributed voter distribution with mean 0 and standard deviation 0.5, equilibrium strategies for the mainstream parties in the two-party, dominant party scenario are constricted by fifty percent, from \( \pm 0.627 \) to \( \pm 0.314 \), and a bit less in the four-party model, from \( \pm 0.427 \) to \( \pm 0.235 \). This movement toward the median is to be expected because super-proportional party influence gives greater value to vote shares relative to position.

\textsuperscript{35} If we interpret the dominant-party model in a plurality-based setting, the super-proportional assumption suggests that larger parties have a disproportionate probability of selection as the winner, a not-unreasonable assumption.

\textsuperscript{36} Thus, the weights representing party influence are given by \( p_j / \sum_i p_i^2 \), so, for example, if vote shares in a four-party model are (0.1, 0.4, 0.4, 0.1), then the weights are approximately (0.03, 0.47, 0.47, 0.03).
Table A.1. Policy-seeking Nash equilibrium assuming **all parties are mobile**, with two-, three-, and four-party contests, assuming a normal electorate.

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 parties</td>
<td></td>
<td>-0.627 (50.0%)</td>
<td>0.627 (50.0%)</td>
<td></td>
</tr>
<tr>
<td>3 parties</td>
<td></td>
<td>-0.777 (38.2%)</td>
<td>0.476 (54.8%)</td>
<td>0.998 (7.0%)</td>
</tr>
<tr>
<td>4 parties</td>
<td>-0.96 (8.5%)</td>
<td>-0.415 (41.5%)</td>
<td>0.415 (41.5%)</td>
<td>0.96 (8.5%)</td>
</tr>
</tbody>
</table>

Note: Voters are normally distributed, with mean 0 and standard deviation 0.5. Values in parentheses are the vote shares at equilibrium. A parliamentary mean model with $q_2 \leq \bar{x} \leq q_3$ or dominant party model with $q_2 \leq x_i$ and $q_3 \geq x_4$ is assumed.
Table A.2. Policy-seeking Nash equilibria for mainstream parties in a four-party model, for a range of extreme party positions; voters are normally distributed, with standard deviation = 0.5.

<table>
<thead>
<tr>
<th>Extremity of extreme parties</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parliamentary mean model with $q_2 \leq \bar{x} \leq q_3$ or dominant party model with $q_2 \leq x_1$ and $q_3 \geq x_4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>± 0.5 (1 s.d.)</td>
<td>0.5 (23.0%)</td>
<td>-0.240 (27.0%)</td>
<td>0.240 (27.0%)</td>
<td>0.5 (23.0%)</td>
</tr>
<tr>
<td>± 1.0 (2 s.d.)</td>
<td>1.0 (7.7%)</td>
<td>-0.427 (42.3%)</td>
<td>0.427 (42.3%)</td>
<td>1.0 (7.7%)</td>
</tr>
<tr>
<td>± 1.5 (3 s.d.)</td>
<td>1.5 (2.1%)</td>
<td>-0.541 (47.9%)</td>
<td>0.541 (47.9%)</td>
<td>1.5 (2.1%)</td>
</tr>
<tr>
<td>Dominant party model with $q_2 &gt; x_1$ and $q_3 &lt; x_4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>± 0.5 (1 s.d.)</td>
<td>$q_2, q_3 = \pm 0.429$</td>
<td>0.5 (17.7%)</td>
<td>-0.429 (32.3%)</td>
<td>0.429 (32.3%)</td>
</tr>
<tr>
<td>± 1.0 (2 s.d.)</td>
<td>$q_2, q_3 = \pm 0.8$</td>
<td>1.0 (6.2%)</td>
<td>-0.540 (43.8%)</td>
<td>0.540 (43.8%)</td>
</tr>
<tr>
<td>± 1.5 (3 s.d.)</td>
<td>$q_2, q_3 = \pm 1.2$</td>
<td>1.5 (1.9%)</td>
<td>-0.586 (48.1%)</td>
<td>0.586 (48.1%)</td>
</tr>
</tbody>
</table>

Notes: Table entries are locations of the mainstream parties $x_2, x_3$ at Nash equilibrium; these parties are assumed to locate in symmetrical positions at equilibrium. Values in parentheses are vote shares for each party at Nash equilibrium.
Figure A1. Values of $\partial U_3 / \partial x_3$ as $x_3$ changes (pictured for the four-party scenario)

a. If $x_3 = 0$, then $\partial U_3 / \partial x_3 \geq 0$.

b. If $x_3 = x_4$, then $\partial U_3 / \partial x_3 \leq 0$. 
Figure A2. Mixed normal voter distribution with extreme Nash equilibria

Note: The mixed-normal voter distribution is specified by $f(x) = 0.5 * f_1(x) + 0.5 * f_2(x)$, where $f_1$ is normal with mean $-\mu$ and standard deviation $\sigma$ and $f_2$ is normal with mean $+\mu$ and standard deviation $\sigma$. In this example, $\sigma = 1$ and $\mu = 1.5$. If in the four-party scenario, $x_2 \approx -3.8$ and $x_4 \approx 3.8$ (and provided that $x_1$ and $x_4$ are more extreme than these mainstream parties), then a Nash equilibrium occurs, but not for more moderate locations for the mainstream parties.